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NUMERALS AND NUMBERS, INDEX NUMBERS AND COEFFICIENTS IN INTER-, MULTI-, TRANS-, AND CROSS-DISCIPLINARY RESEARCH'S LANGUAGE

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Abstract. *Numerals (digits), numbers, index numbers and coefficients are sometimes considered some usual mathematical expressions from a fundamental scientific language but also elements of the modern inter-, multi-, trans-, and cross-disciplinary research. In their first signification, numerals (digits), numbers, index numbers (IN) and coefficients are the result of a long process of evolution and a vast typology is a key to understanding all of their senses. In their second signification numerals (digits), numbers, index numbers and coefficients are the components of modern research and education based on an inter-, multi-, trans-, or cross-disciplinary approach. After a brief introduction underlying three vital questions for any researcher, this paper details the numerous family of numerals (digits), numbers, index numbers and coefficients, with some similarities and differences in their usual sense in mathematics and in any inter-, multi-, trans-, and cross-disciplinary research. Some final remarks are important for a better understanding of mathematical language in modern research.*

Keywords: *quantity, measure, numeral (digit), number, index number (IN), coefficient, elasticity, correlation, associations, inter-, multi-, trans-, and cross-disciplinarity.*

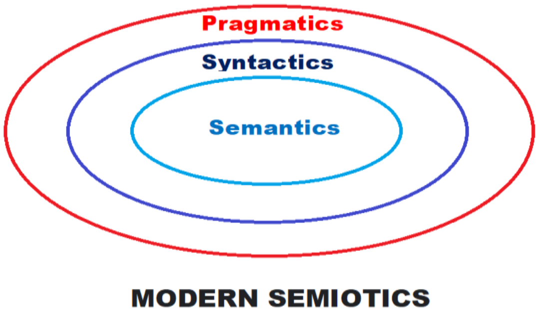
1. INTRODUCTION

During a quantitative scientific investigation of socio-economic or demographic real life, in a usual or even unusual inquiry, where organizing observed data into meaningful structures becomes essential, researchers must answer the minimum next three major questions about numerals (digits), numbers, index numbers and coefficients: I) Can one research be finalized as a real investigation without defining, redefining and even underlining the most important differences between numerals (digits) and numbers or between index numbers and coefficients, and especially without a methodology of practical validation of these primary instruments etc.? II) Can the same research offer a qualitative conclusion or a final remark without the help of measured variables, or without some analyses based on associations or correlations between the same phenomenon's variables? III) Can researchers obtain some reasonable or valid results and especially a short-term foresight without some specific methods or models, based on using sometimes even only a few numerals (digits), numbers, index numbers and coefficients? These questions and their specific answers perturb the way of thinking of any researcher and he or she must find an answer before starting his or her new research.

One can now imagine these expanded aspects in inter-, multi-, trans-, and cross-disciplinary research when all these are multiplied and hence significantly different in importance and impact. In this way, the introductory approach of this article can be considered already a complete one. In the central section of the paper, the necessary notions are first all defined and detailed to specific inter-, multi-, trans-, and cross-disciplinary research. Finally, all the researchers must be connected to the whole investigation in a characteristic team system, and the results many times expressed in numerals and numbers, index numbers and coefficients. Some final remarks or conclusions are drawn in the end, emphasizing the main gains of quantifying results and validating or not initial hypotheses, thus developing a wider or better understanding of reality.

2. NUMERALS (DIGITS) AND NUMBERS' EVOLUTIONS AS ESSENTIAL TERMS IN MATHEMATICAL LANGUAGE

What really are numerals (digits), numbers, index numbers and coefficients in Mathematics results or in its scientific language? Semiotics, born from "*semesion*" or sign, in the Greek language offers a credible answer to this question [1]. This answer is only the word "*signs*". Semiotics, as modern science, developed from the study of meaning in language, can be applied to entire text or only to a single word. In the same old Greek language "*semainon*" was used as a signifier and "*semainomenon*" meant signified or indication. In modern Semiotics are reunited Semantics, Syntactics (Syntax), and Pragmatics (fig. no.1):



Source: Realized by author

Fig. 1. Structure of Modern Semiotics - base layers

Semantics means the associations of signs to what they stand for, meanwhile Syntactis offers formal or structural relations between the same signs, and Pragmatics underlines the link of signs to interpreters. The initiators of this difficult and more and more extended science, named Semiotics remain Ferdinand de Saussure [2], and Charles Sanders Peirce [3]. The first, a well-known linguist from Switzerland, describes the new science as a theory of sign and "baptize" it as Semiology. Saussure's Semiology is based on the principle that "*emphasized language as a system of sign, and besides language, there are many other sign systems that exist in the world of mankind*." [2]. The second scientist was an American philosopher well-known as a pioneer of pragmatism doctrine, who named Semiotics the new science"*synonymous with the concept of logic that focuses on the knowledge of human thinking process* [3]," and thus main principles containing Peirce's theory, were *"the human mind and sign boundaries, the three-dimensional system (triadic/trichotomy) and the relativity regarding the three typologies or taxonomies of signs: i) icon; ii) index,* [including Index Number–IN and Coefficient-C]*; iii) symbol* [3]."

But science's evolution is more complicated and sometimes a simplifying personality must solve or complicate everything. This was the case with Umberto Eco for Semiotics. In his *Theory of Semiotics*, Umberto Eco is connected with all aspects that can be taken as a sign, "*generalized as everything which can be taken as significantly substituting for something else*" [4]. Sometimes that "*something else*" does not exist exactly at the same time when the sign represents or replaces its position, and therefore, Umberto Eco often refers to the *Theory of Semiotics* as a theory of lie (deception) because it can be used for misleading or deceiving the others [4].

Can a researcher say about numerals (digits), numbers, index numbers and coefficients are signs used for misleading or deceiving some other researchers? Well, in Mathematics' results or in its scientific language when this aspect appears soon or later must be present a classic *erratum* or a calculus must be declared affected by real error …

But all definitions can clarify better any of the concepts, including numerals (digits), numbers, index numbers and coefficients in mathematics. Numerals are just simple "*symbols or collections of symbols used to represent small numbers, together with systems of rules for representing larger numbers*." [5] Are numerals real numbers or are they not? The earliest numerals were either simple notches on a stick or rudimentary scratches on a stone, marks on a pottery piece etc. There was no real necessity for written numerals until the beginning of civilized times. Initial, the first numeral designates a number of objects being necessary as a symbol for the small numbers, and thus for abstract notions like one or two humans someone needs independent signs, and particular representations and all of these probably appeared very late [6].

Numbers are complex constructions if one tries to compare numbers with numerals. Numerals are the original bricks for all these constructions named numbers, together with some rules for obtaining a correct solution on different landscapes or streets, and without restriction to the decimal system. After these rules redefine words as digit and numeral in any base (binary, senary, octal, decimal, hexadecimal, etc) and any systems of numeration (the Hindu-Arabic system, the Babylonian system, the Greek system, the Chinese system), anyone can build numbers from numerals (digits).

The first numbers were usual arithmetic values used for representing the quantity**,** but all evolved in measuring, in making calculations, in counting, in keeping things in order, in indexing, etc. Initial or innovative numbers need numerals (digits or symbols in a logical manner) or a specific number system. The first numbers were natural, extracted from nature. Beyond the natural numbers, there are other types of numbers that have emerged throughout history in order to express situations and solve problems for which the first were insufficient [7].

From a semantical point of view, between numerals (digits) and numbers, there is not only one connection but also there are different meanings. Thus, numbers sense can be similar to "semainon", being used as a signifier, and numerals (digits) meaning sometimes could be signified or indication ("semainomenon"). A long period, in the old Greek mathematic language and another even longer period after them, "numbers" were just integers, constructed from "numerals" (digits), from 1 to 9, not including 0 ("zero", "sunya", "cipher", "sifr", "zephyrum" etc.). Zero was taken from the Babylonian origin in the Indian mathematic language and somehow purified through Chinese and Egyptian logic and pragmatic minds. "Zero" became a very important or special numeral, defining not only the absence of all sizes or quantities but also a unique landmark or a specific border placed between all negative numbers and all positive numbers. Finally, zero is a symbol used to represent the absence or the emptiness.

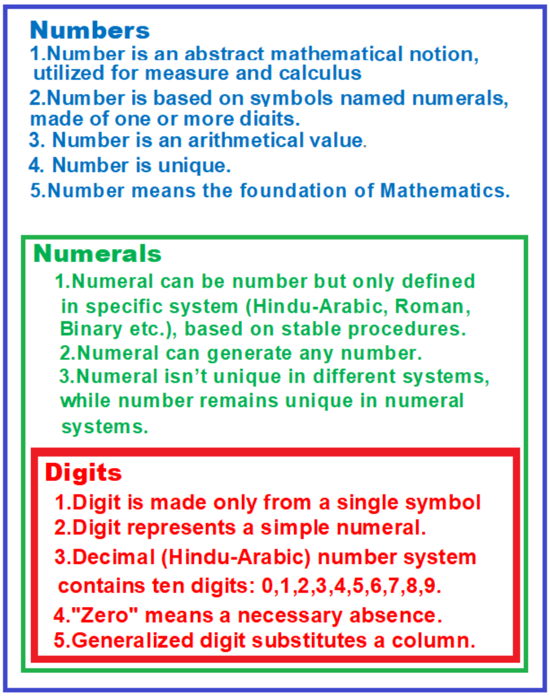
Without a doubt, all of the researchers can remember integers, rational numbers, real numbers, in fact following numbers evolutions to integer, or to remain rational and why not to maintain reality for a brief or a long period of time. In the case of real numbers, it means probably a too long period of time: "*Euclid and Archimedes had together a good intuitive grasp of the concept of real number, and they had a theory of proportions when discussing such things as lengths and areas, and* *what we call irrational numbers. This theory of proportions was used up to the 18th century"* [1].

But what means time in mathematics language evolution and especially in the theory of numbers and as a proof, how many of us really know that there are newborn numbers [7] baptized as "*happy numbers*" or "*palindromic numbers*"?

The major difference between a numeral (digit) and a number remains that a numeral (digit) is a single numerical symbol, whereas a number can contain a single numeral (digit) or a combination of more numerals (digits). Anyone can try immediately another question like: What's the difference between a numeral and a digit? It might look easy to answer it, but in truth, it's indeed a very difficult way to do it, without mathematical rules. Digit in the dictionary is always defined as approximately unclear like: "*any one of the ten numbers 0 to 9"* (*Cambridge Academic Dictionary: Cambridge University Press,* and *the same digit disappears and* becomes a little detailed as an Arabic numeral in Merriam-Webster: "*any of the number symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9*" [8]. Some mathematical dictionaries use other definitions as: "*A digit is a symbol (0, 1, 2, 3, 4, 5, 6, 7, 8, and 9) used to write the numbers.*" (*Icoachmath.com Math_dictionary*) [9] The basic definition of digits became somehow a similarity with numerals, sometimes a profound one, between these two mathematical terms: *"any of the numerals from 0 to 9, especially when forming part of a number"* or "*elements used in forming numbers in other number systems*" [10]. It means one can use a group of numerals for written digits in a number, based on a mathematical system, named *decimal system of numerals* (nine digits and a zero). The first synonym for digit still remains **numeral**, followed in a strange manner by *number*, figure, integer etc. That is because the digit's notion implies duality, "*modern digit*" having the meaning when of a number and when of a column defined through ranges of equal value (i.e. "7-digit" incomes as a simplified expression for very high salaries and thus describing how many columns someone really needs to write the number and not a real wage).

"Modern digit" is still a numeral or rather a symbol and less a number, having the quality of a substitute, emphasizing a numeric value level and changes, anytime the same symbol is put in a different place when someone writes numbers.

From the beginning, a number transformed a measurement or a quantitative idea from human minds into reality. During the last three or four millennia, the entire humanity used numbers to think, discuss or write about almost everything, the numbers being usual, unusual or even unique like π and ε, the special number being placed always behind mathematical ideas. The most important association between number and numeral (digit) remains that of their mandatory coexistence in mathematical language. Thus, a mathematical idea, called a number, can become a written reality only with the help of a numeral (digit). Dominant relativity, as the law of numerals' or digits' association, generated a type of continuum of symbols, between idea and reality, but also reciprocally. Similar to letters making words in literature, numerals (digits) stand for any idea of a number as an excerpt initially from reality, and finally from a mathematical calculus (algorithm etc.). From the Latin language, the digit was translated with a similar sense of a finger in mathematical language. Romans were also those who tried really hard to unify symbols from the Latin language and used letters as substitutes for numerals (digits). But as with many other unsuccessful attempts, a generalized compromise was again entirely refused by mathematical language throughout its history. Numerals (digits) and numbers remain even now the "*foundation of mathematics*" and the theory of numbers became the "*queen*" of mathematical theories [7]. A synthesis of the most important aspects characterizing numerals (digits) and finally numbers is detailed below (fig. no. 2):

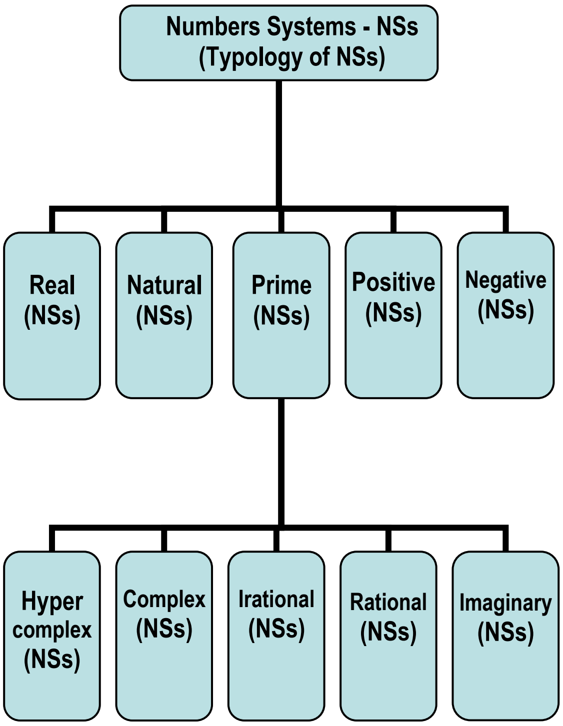


Source: Realized by author.

Fig. 2. Structure of mathematical numbers

The mathematical language was continuously enriched and diversified by the typology of numbers, which has developed a family into more and more complex numbers (even hypercomplex numbers). Typology has become more important for numbers than arithmetic operations.

In fig. no. 3 some of the new classes of numbers are several classes of numbers that are detailed with a purely demonstrative role as a proof of a continuous introspection in the complex universe of the theory of numbers, extended from numeral (digit) to number, regrouped criterionically in systems and under the influence of inter-, multi-, trans-, and cross-disciplinary researches:



Source: Realized by author
Fig. 3. Typology of Number Siystems (NSs)

In the modern language of mathematics, any real number from any NSs can be described as an (in)finite set of numerals (digits) or as a numeral's sequence, where the uniform distribution becomes closer to reality, and the natural density tends to be approximately equal anywhere from algebraic statistics to financial econometrics [11, 12].

3.INDEX NUMBERS(INs) AND COEFFICIENTS (Cs) AS RESULTS FROM INTER-, MULTI-, TRANS-, AND CROSS-DISCIPLINARITY

Index Numbers (INs) and Coefficients (Cs) are coming not only from Peirce's theory of the *"three typologies of signs: i) icon; ii) index* [INs & Cs]; *iii) symbol* [3]," but also from inter-, multi-, trans-, and cross-disciplinary researches [13]. The modern numbers' diversification is based more on the experience from these four research's attempts or forms of crossing the borders of any isolated discipline or any unidisciplinary attitude: inter-, multi-, trans-, and cross-disciplinarity, broadening the perspective of knowledge.

In any multi-disciplinary research "*the subject under study is approached from different angles, using different disciplinary perspectives*," multiplying results and implicitly numbers, but even so "*neither the theoretical perspectives nor the findings of the various disciplines are integrated into the end*" [14], meanwhile any interdisciplinary research "*creates its own theoretical, conceptual and methodological identity, and as consequence even its results* [including numbers] *in an interdisciplinary study being more coherent and integrated*" [14]. Any multidisciplinary research remains within the boundaries of initially connected disciplines, based on their specific knowledge and way of thinking. while any interdisciplinary investigation "*analyzes, synthesizes and harmonizes links between disciplines into a coordinated and coherent whole*" [15].

Cross-disciplinarity can generate also new numbers or classes of numbers by applying methods from a well-known science in various other newer or not sciences or confers values initially unsuspected to some numbers by changing the domain, through the specific measurement of new phenomena, based on the units of measure or even because of some pragmatic details.

Starting from its own three methodological axioms, and in conformity with Basarab Nicolescu's books and papers, trans-disciplinarity multiply essential numbers and enrich also their typology, having access to: "*different levels of Reality of the Object and of the Subject (ontological axiom), also to the passage from one level of Reality to another, ensured by the logic of the included middle (logical axiom), and finally to the structure of the totality of levels of Reality, as complex structure, where every level is what it is because all the levels exist at the same time (complexity axiom)*" [16, 17, 18, 20].

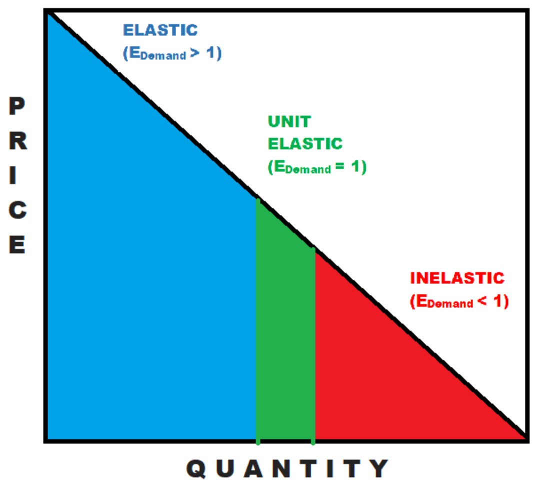
Index Numbers (INs) and Coefficients (Cs) are the real expressions of modern multiplying sense or significations of numbers in the mathematical language under the influence of inter-, multi-, trans- and cross-disciplinarity. Index numbers (INs) are the measures of average (overall) changes in a group of related variables, having a constructive role in homogenization in many heterogeneous statistical populations. You cannot value an average of values placed in extremes, such as the values of the prices of different products on a global market, but when you use indices, this population of the resulted indices suddenly becomes homogeneous and close to normal and continuous distributions

INs appeared more than three centuries ago, "*from the remarkable curiosity of an English cleric and political arithmetician, William Fleetwood, to measure the real evolution of the purchasing power of the population in England, which is research published in Chronicum preciosum*" [20]. INs have gradually occupied the new interdisciplinary and multidisciplinary researches in economics, health, sociology, demography, management etc. In Statistics, INs forms a specific method of analysis and covered more and more domains, based on cross-disciplinarity. Transdisciplinary approaches based on INs are useful as solutions for transforming any heterogeneous population or database into a homogeneous one because of their capacity of measuring flexibility in time, space or structure, not only more properly, but also more precise. INs are indeed modern numbers because can guarantee that certain conditions will be better fulfilled in general research like the ideas of stability, order, balance, substitution, hierarchy, aggregation, disaggregation, and thus generating the concept of INs‘ system, These new statistical numbers named INs can measure now almost everything and everywhere.

Coefficient as a mathematical notion was introduced by French mathematician François Viète (1540-1603), influenced by Modern Latin, after a brief period when it has been initially written as "*co-efficient*" and represented "*a number or a letter which unites in action with something else to produce a given effect,"* The first sense of the concept was "*acting in the union to the same end*" [21]

The coefficient is a numerical or constant quantity placed before and multiplied by the variable in an algebraic expression, and a modern number helps the researcher to valorize a measure of some property or characteristic (as of a substance, device, or process). Any coefficient underlines if the value of an independent variable tends to increase/decrease, being useful in many investigations, referring to economic elasticity, industrial flexibility and rigidity, metal expansion, function value, etc.

Any coefficient as any number is unique but only up to a scaling operation, so the magnitude of some newborn coefficients can remain arbitrary or exposed to errors or omissions (i.e. final limits in elasticity of demand in fig. no. 4)



Source: Realized by author.

Fig. 4. An example of Cs' evolution and limits

The difference between the two initially exclusively spatial, temporal and structural statistical constructions called classical IN (i.e. Laspeyres or Paasche type) can be found during a mathematical demonstration in the very essence of the elasticity coefficient. Thus, statistical thinking passes mathematically from the INs method to the Coefficient (Cs) method of elasticity [22].

The relationship between the index and the coefficient identifies the essence of the phenomena represented, namely the economic elasticity (of demand or supply) and represents a statistical correlation between two pure INs' and Cs' methods, using the new numbers as original interrogative and demonstrative cycles of cross-disciplinary thinking [22, 23]. If the index (under the name Index-Number) was used to measure a change of a quantitative nature, which could not be directly observed (Bowley, 1920) or to show either the increase or the decrease of a fairly large value difficult to measure accurately (Edgeworth, 1925), the INs Method was and is an insufficiently exploited method directly and the encounter with the elasticity coefficient method was not accidental (Bortkiewicz, 1932), but represented a creative association that generated both new numbers as well as original inter-, multi- and transdisciplinary solutions for their generation [24,25].

4. SOME FINAL REMARKS

Day after day, the language of mathematics develops more and more from numeral (digit) to numbers, and more recently, through "*happy numbers*" or "*palindromic numbers*", it gets closer and closer to the richness of any language. To have a more comprehensive view of the future of numbers in mathematical science, and their cohesion or unity as a universe structured by systems, sets, classes, groups etc. and multiplied through inter-, multi-, trans-, and cross-, disciplinarity we need many more theoretical researches and practical investigations. However, even this paper can be a good example, showing that overcoming the fragmentation created through scientific questions remains a valuable solution that requires deep foundations and overall evolution analysis, especially in the form of more clearly defined mathematical terms in mathematical language. "Quantitative description of phenomena, perceived by Auguste Comte and many others after him as evidence for exactness, can impress only novices" [13], but I believe that without exactness or precision and stable results or numbers there is no mathematical science or mathematical language.

Of course, the objective key will remain forever in the balance between mathematical ontology and mathematical epistemology, and also in the correspondence between language structures that define and describe entities and parts of the whole (logical, and mathematical). I think that more papers and discussions about mathematical language and major mathematical concepts must be experienced, promoted, encouraged, published, integrated into research teams, and exchanged between the same research teams offering more flexibility, selectivity, exploratory analysis, innovative value, even creativity, more adequate scientific vision, relevancy, etc.

As the point elasticity or the arc elasticity can vary in value depending on the starting point, always numbers can evolve in different systems, sets, and classes of high or low volumes, so the numbers will forever remain unique for any mathematician.

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REVIEW OF THE GRAVITY MODEL: A PHYSICAL APPROACH TO SOCIAL SCIENCES

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Abstract. *This article presents a bibliographic review of the gravitational model in sociophysics since its origins in Newton's conception. Considering the crucial role played by physics in the expansion of our understanding about social sciences, I will preserve such an approach throughout the article. Firstly, I will introduce its theoretical roots with an overview of the physical models by the most prominent scholars, up to the recent applications in migration and economics. Secondly, I will illustrate the analogies and differences between the gravitational model and Tobler's first law of geography. Thus, this work results in a sound comparison between a simple spatial framework in three variables, the most notable of which is distance, and a structured scheme based on the gravity metaphor which relies on the concepts of attractions' forces and spatial concentration. Thirdly, I will propose an assessment of the most recent application to intangible goods, as in the fields of marketing and services.*

Keywords: *gravity model, demography, migration, sociophysics, economic geography, social sciences*

1. INTRODUCTION

In physics one of the universal forces regulating the entire universe and acting as a fundamental condition of all our movements is gravity. Such concept has also been successfully applied to social sciences, under the name of 'gravity model'. This article, therefore, is intended to be a historical review of the origins and applications of the gravitational model to social sciences, with a focus on its roots in physics. Whilst it should be mentioned that there is a similar review of thirty years old academic literature on the topic (Philbrick, 1973), the present study represents a step forward, for it offers an updated and detailed analysis with a broader range of the existing applications of the model.

Whilst filling a void in the academic panorama, the study is suggesting an updated thematic division of the gravity model applications to social sciences which can clarify its socio-physical evolution.

This article starts by examining the principles of the gravitational model, from Isaac Newton's formula to its recent applications in the broader field of social sciences, including international trade. In order to respect the mathematical and physical origins of this model, the initial approach rooted in physics and mathematics will be preserved throughout the bibliographic review.

In the consecutive pages, an attempt will be made to answer the following key question: what are the main features of the developments of the gravity model in social sciences? The structure of the review is organised in two paragraphs. The former aims at introducing the main pillars of gravity in physics which are necessary to understand the application of gravity to social sciences, given the physics-based origins of the model. The latter ranges from the theoretical foundations of this application to the review of the different uses of gravity in social sciences, such as in the study of people's movements. In this sense, the Tobler's Law becomes an indispensable point of reference to grasp the theoretical evolution of the gravity model[1].

As aforementioned, this article introduces an innovative approach because its purpose is to trace the theoretical applications of the gravity model over time and to analyse in detail its contribution to social sciences with the aim of achieving a deeper understanding of human behaviour and social phenomena. Moreover, the element of novelty consists in the suitable combination of the sequential scheme with a thematic division into subparagraphs.

I will follow a chronological order, which is the most appropriate to describe the interdisciplinary development of the application of the physical model. Against this background, three historical evolutions emerged with respect to the applications of gravity to social sciences:

- Application to people's movements.
- The shift from people's movements to the interest in flows of things (economic flows); and
- A recent interest in less tangible flows (ideas, services, et cetera).

[1] For example, it would be difficult to fully understand Isard's gravity model without the prior study of Stewart's.

1.1 Isaac Newton's Law

The gravity model owes its name to the homonymous law in physics.[2] The discovery of gravity law is not due to an individual work but it is the result of the contributions of several scientists.[3] As famously stated by Newton himself: "If I have seen further, it is by standing on the shoulders of Giants" (Newton 1675). The Earth exerts a force of attraction towards the ground (as in the case of the apple falling from the tree). Newton realized that this rule was not only valid on the surface of the Earth but was equally applicable throughout the universe (universal law), being directly related to the mass of thing, of both terrestrial and celestial bodies. Johannes Kepler (1609) had previously described how planets move through space following elliptical orbits.[4] In addition to demonstrating the accuracy of this model, Newton further developed Kepler's Third Law of planetary motion, which described how planets move in ellipses with the sun always at one focus of the ellipse. Therefore, for instance, the same force was responsible for keeping the Moon in Earth's orbit. This law is defined in physics with the famous formula of the Law of Universal Gravitation proposed by Newton (1687). With respect to the terminology used, the word 'gravity' derives from Latin gravitas, meaning heaviness (weight); its first appearance in a scientific text, written in Latin, dates back to the book Philosophiae Naturalis Principia Mathematica, published in 1685 (Schmitz 2018: 251). This law is universal, and it postulates an invisible force of attraction of matter which acts through both long and short distances. Indeed, in the universe two particles of matter attract each other with a force directly proportional to the product of the masses and inversely proportional to the squares of the distance between them. The closer the bodies are and the bigger their masses, the higher their attraction is to each other. In the formulation of this law, Newton created the concept of mass, that is, the quantity of matter of a body (volume by density), which was distinguished from that of weight, which varies according to the acceleration of gravity. The measure of weight changes in space, while mass is a fixed quantity which does not vary. The larger the mass of a body, the more powerful the force of gravity. The formula developed by Newton is the following (Schutz 2003: 13-18):

$$F_{ij} = G \; \frac{M_i M_j}{D_{ij}^2} \qquad (1)$$

F_{ij} is the attractive force, M_i and M_j are the masses, D_{ij} is the distance between the centres of the two objects, G is a gravitational constant (universal gravitational constant[5]) depending on the unit of measurement of mass and force, the subscripts i and j indicate the pair of bodies considered.[6] The force will have the same direction as the intersection line joining both material points; bodies with spherical mass distribution attract and are attracted as if all their mass was located in their centres.[7]

Since gravitational attraction is expressed as a force in physics, its vector form is important. Indeed, force manifests itself in the mutual interaction of two or more bodies, which vary their state of rest or uniform motion, it is expressed as a vector quantity because it does depend on which direction that force is applied.[8] Considering its effect on one of the bodies, its vector will be composed of a magnitude (also called length or norm), a directed axis and a direction (also called versor, unit vector or orientation). The applied intensity will be its magnitude, which is exerted between two points in space (directed axis) and from one point to another (direction).[9] In the presence of multiple bodies and

[2] Unless otherwise specified, all the contents presented are based on the following textbooks, as indicated among the bibliographic resources: James B. Hartle, Gravity: an introduction to Einstein's general relativity (London: Pearson, 2003); Bernard Schutz, Gravity from the ground up - An Introductory Guide to Gravity and General Relativity (Cambridge: Cambridge University Press, 2003), 9; Charles W. Misner, Kip S. Thorne, and John A. Wheeler, Gravitation (Freeman, 2000).
The reader unfamiliar with this field of studies is referred to these books, or to any other introductory textbook, for a better explanation of the concepts and formulations used in this paragraph.

3 Among these, Kepler (1571-1630), Galileo (1564-1642), Descartes (1596-1650), Borelli (1608-1679), Huygens (1629-1695), Robert Hooke (1635-1703). Their studies laid the foundation of Newton's law of universal gravitation contained in the Principia (Jourdain 1913:353-384).

4 Kepler explained the motion of celestial bodies with its three laws. The first law stated that the planets describe around the sun elliptical orbits where the sun represented one of the focal points. The second law held that the motion of the planets was uniform. The third law stated that the squares of the planets' motion times to describe the orbits were proportional to the cube of the length of the same major axis of its orbit.

5 It is called also Big G for disambiguation with the local gravitational field of the Earth (small g). Newton did not know the precise value of G but tried to approximate it. Its value is difficult to measure because it is weak as compared to the other force, but it acquires relevance for celestial bodies' masses. It is approximately 6,67 x 10-11 N m2 kg -2 (that is newton per square meter divided by square kilogram).
Being small g the acceleration, equal to 9,8 m/s2 on the Earth (that is meter divided by square second), we will have that, for the object i, g= (GM_j)⁄(D_ij^2).

6 Force is measured in newton (N), masses are measured in grams (g), and distance in meters (m).

7 The Shell Theorem formulated by Newton in Principia, probably inspired by Robert Hooke's theories of centripetal gravity of celestial bodies, states that a spherically symmetric mass, having uniform density, exerts a force on the outside as though all its mass was concentrated at its center, while exerting a zero force inside.

8 In physics, we distinguish between scalar quantity and vector quantity. The former is a quantity described solely by a real number (magnitude) that does not depend on the direction, while the latter has a magnitude and a direction (which technically is the ensemble of directed axis and orientation). Common vector quantities are force, velocity and acceleration. Common scalar quantities are distance, speed and energy.

9 A vector, metaphorically like an arrow, with a certain intensity (magnitude) will follow a trajectory along its rod (directed axis)

their respective attraction forces, the force deriving at a point will be the result of the vector composition of the attraction forces considered (vector sum[10]). The vector form is the same of the equation (1), except that F is now a vector and the right-hand side is multiplied by the unit vector from object i to object j.

$$\vec{F}_{ij} = -G \frac{M_i M_j}{|\vec{D}_{ij}|^2} \frac{\vec{D}_{ij}}{|\vec{D}_{ij}|} \quad (2)$$

$\vec{F}_{ij}$ is the attractive force applied to object j and exerted by object i, M_i and M_j are the masses, $|D_{ij}| = |D_j - D_i|$ is the distance between the centres of the two objects, G is a constant gravitational (universal gravitational constant), $\frac{\vec{D}_{ij}}{|\vec{D}_{ij}|} = \frac{D_j - D_i}{|D_j - D_i|}$ is the unit vector from object i to object j. In the vector form, we will have that $F_{ij} = -F_{ji}$.

In ancient times, it was thought that forces originated exclusively because of contact.[11] Instead, according to the law of universal gravitation, two objects can interact with each other through instantaneous action at a distance. Newton observed the phenomenon of gravitational attraction in various experiments and demonstrated it mathematically, without knowing its cause (hence his famous quote "hypotheses non fingo").[12] Under the pressure of the 19th century's discoveries by Faraday (Ulaby and Ravaioli 2007: 255) and Maxwell in the context of Electromagnetism,[13] the concept of a field was introduced, which, unlike the Newtonian idea of instantaneous interaction between bodies, identified a set of values that a physical quantity assumed in a region of the space. Ergo, space ceased to be purely geometric: it expressed quantity as a function of position. In this sense, gravity can be described through a vector field in which the magnitude that varies is the force of gravity. This field can be represented graphically through an aura of force lines that from every point in space are directed towards the generating mass' centre and that will be more intense near the surface of the mass. In this regard, the gravitational field equation is defined as the ratio of the force of gravity $\vec{F}$ acting on the mass m, where the gravitational field vector $\vec{g}$ is measured as an acceleration (m/s^2) and is parallel and with the same direction/orientation of the force[14].

$$\vec{g} = \frac{\vec{F}}{m} \quad (3)$$

Considering the intensity of the gravitational field in a single point with distance r from the generating mass' centre of the field, as in the case of the Earth exerting a gravitational field on a mass located at that point, there will be an oriented force towards the centre of the earth equal to:

$$\vec{g} = \frac{G\,M}{D^2} \quad (4)$$

The intensity of the field will depend on the size of the mass generating it and the distance (D).[15]

The energy associated with the mutual influence of two gravitational fields interacting with each other is called Gravitational Potential Energy, namely the work against gravity performed in moving the mass of an object to a given point in the gravitational field of another body and it is ready to be transformed into another form of energy.[16] Its formula is:

$$U = -G \frac{M\,m}{D} \quad (5)$$

where U is the gravitational potential energy[17], G is the gravitation constant, M is the mass of the attracting body, m is the mass of the attracted body, and D is the distance between their centres. More commonly, M represents the mass of the earth. The negative sign indicates the energy expenditure of the body m moving in the attracted direction.
Considering m as a point unit[18] we will get the gravitational potential (V), which does not depend

proceeding from the arrow's nock towards its tip (direction or versor).

10 The interested reader will be able to further explore the rules of vector sum in any elementary manual of physics or engineering. I suggest Bruce R. Kusse and Erik A. Westwig, Mathematical Physics: Applied Mathematics for Scientists and Engineers, Second Edition (Weinheim: WILEY-VCH, 2006), 1-17 and 44-58.

11 That is in physics a push, a traction or a shock.

12 For Newton, it is not possible to speculate on something that cannot be proven. In Principia, we read: "I have not as yet been able to discover the reason for these properties of gravity". For a rational explanation of the cause of gravity, we have to wait until 1915, when Albert Einstein presented the Theory of general relativity to the Berlin-Brandenburg Academy of Sciences and Humanities.

13 It is worth noting that Maxwell's equations for electromagnetism have been called the second great unification in physics, where the first one was realized by Isaac Newton (Feynman et al. 2011, cap.18) and the third will be realized by Einstein.

14 See supra note 10.

15 In the terrestrial case for r equal to the Earth's radius, it results that the acceleration of gravity on the Earth's surface is equal to 9.8 m/s2.

16 More specifically, potential energy is stored energy. The body of an object is endowed with it due to its position or orientation.

17 Another more general form considers the distance from the earth (h) and the Earth's gravitational acceleration ($g = 9.8$ m/s^2). Its formula is $U = gmh$.

18 Inversely, considering M as a point unit we will obtain the gravitational potential (V) that does not depend on the possible mass quantity placed in it. The gravitational potential which the mass m produces at that point of the attractive mass (V_M) will be therefore $-G\frac{m}{D}$.

on the potential quantity of mass placed in it. The gravitational potential produced by the mass M at that point of the attractive mass[19] will therefore be equal to:

$$V_m = -G\frac{M}{D} \tag{6}$$

2. THE CONCEPT OF GRAVITY IN SOCIAL SCIENCES

2.1 Theoretical Foundations

Among the earliest applications of science to social phenomena[20] the study by the astronomer Edmond Halley (1693) should be recalled.[21] He calculated the "just value to be paid for an Annuity during the whole term of the Lives" combining demographic concepts with the calculation of probabilities for risk assessment.[22] However, the concept of social physics as such only emerged in the 19th century, starting with the Lettres d'un habitant de Genève à ses contemporains (1803) by the French philosopher and economist Saint-Simone. This work was a tribute to science, provocatively seen as a new religion devoted to Newton's cult and aimed at the union of the most important scholars in the world.[23] The French philosopher aspired to the integration of the scientific principles of different fields of study through a revolution of Western and European thought, which had to put science at its centre. Thus, the scholar had the idea of describing society using laws such as those of physics and biology. However, it was his pupil Auguste Comte (1854: 39) who first used the term social physics, defining it as "that science which occupies itself with social phenomena, considered in the same light as astronomical, physical, chemical, and physiological phenomena, that is to say as being subject to natural and invariable laws, the discovery of which is the special object of its researches". Later, the Belgian astronomer Adolphe Quetelet (1835) resumed the term social physics in his work A Treatise on Man and the Development of his Faculties[24], where he analysed society through statistical probability[25], comparing the individually measured observations with their normal distribution, which was obtained from a sample. Indeed, through the application of the scientific method to the analysis of human phenomena the astronomer outlined what today we would call the social sciences. When Quetelet adopted the term social physics, Comte coined the word sociology to circumscribe his field of study[26], perhaps for reasons of rivalry (Jahoda 2015).

However, over time social physics acquired the meaning of a more specific field of study aimed at the application of physics to the social sciences, while according to Comte, sociology referred to the science collecting all studies on humans (such as history, economics, psychology, et cetera).[27] Indeed, social physics (also called sociophysics) is defined today as an interdisciplinary field of study that uses mathematical tools developed in physics to deepen our understanding of social phenomena and human behaviours. Among these, the concept of gravity has been widely applied to social sciences' studies. Models based on this force of attraction are part of a large category of frameworks, which are described as spatial interaction models.[28] Their formulas are mainly, yet metaphorically, based on Newton's law of universal gravitation (1) or on the gravitational field's equation (4). These functional forms have been applied to a whole series of 'social interactions', such as migrations[29], tourism, trade, foreign direct investment, demographics, cultural exchanges and more. More generally, the application of the gravity model to social sciences predicts a flow between partners (countries, regions, companies, or other subjects) in an intuitive way.[30] Drawing on gravity's laws application to social

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[19] When there are more than two masses in a given space, the total potential will be equal to the sum of the bilateral potentials

20 The interested reader will be able to explore more in-depth some scientific applications to the field of social sciences before Halley's ones in: Gianfranco Tusset, From Galileo to modern economics: The Italian Origins of Econophysics (Springer International Publishing, 2018).

[21] Halley's most important discoveries include identification of solar warming as the cause of atmospheric motions, the relationship between barometric pressure and sea level height, the calculation of an elliptical orbit for a visible comet, now known as Halley's comet. Halley was also interested in the gravity and proof of Kepler's laws of planetary motion, for which he financed and supported his colleague and friend Newton. Newton published the book Philosophiae Naturalis Principia Mathematica thanks to Halley's funding.

[22] This provides the theoretical basis for the future birth of the life insurance sector.

[23] This idea will be better explained in 1814 by Saint-Simone in "Réorganisation de la société européenne".

[24] The original title in French is "Sur l'homme et le dévelopment de ses facultés, ou Essai de physique sociale".

[25] The scholar collected statistics on madness, drunkenness, crime and suicides by establishing relationships with some predictive variables.

[26] Comte used the term 'sociology' because he disagreed with Quetelet's ideas that a theory of society could be the result of a collection of statistics.

[27] Although, as already mentioned, the terms social physics, sociology and social sciences were coined in the same period, and at the time they were often used with the same value and meaning, they have acquired over time different nuances. In particular, social sciences are intended in a broader sense that differs from the natural sciences, since the former (social sciences) focus on human activity. Sociology, instead, is a specific social science that studies society and its relationship with the individual.

[28] According to geographer Peter Gould (1958:57), Geographic theory "is a matter of gravity".

Finally, social physics refers to the application of physics and its rules (physics is a natural science) to the social sciences.

[29] See the gravity model of migrations in Section 2.3.

[30] Detractors of the gravitational model accuse it of being only based on observations, in addition to being biased towards the largest centres and grounded on historic ties.

sciences, the aforementioned flow is attracted in proportion to a specific measure of size and proximity, depending on frictions of the distance, mainly expressed in terms of distance, time or cost. According to economist Walter Isard (1954: 308) "the distance variables act in much the same manner with respect to the social world as to the natural world". Therefore, a gravitational relationship must arise in any model that considers distance to be a direct factor that implies costs. Carrothers (1955: 99) argued that the analogy between human interaction and Newtonian physics of matter was possible. Although "it may not be possible to describe the actions and reactions of the individual human in mathematical terms", the behaviour of a set of people was instead predictable thanks to the study of probability in mathematics. According to the author, this phenomenon was observable in all the social sciences, since people behave differently in groups than they do as individuals.

Similarly, in physics it was not possible to describe the behaviour of molecules taken individually, while it was possible to predict the action of a set of molecules. The logical considerations just mentioned, in particular those of Compte (1854), Isard (1954), and Carrothes (1955), represent the necessary premises to develop a physic reasoning in the social sciences, and without which a gravitational sociophysical structure would make no sense, due to the lack of logical coherence/consistency. In a certain sense, one is free to believe or not that human actions are guided by universal laws. Nevertheless, beyond the solution to this complex dilemma, one is obliged to accept a sort of determinism in human beings' life, if one wants to use a gravity model in the social sciences.

In social sciences, the first component considered is the attribute's size[31] (quantitative variable), which is positively related to the phenomena under examination (the formula's result which is usually a flow). On the contrary, the second element, distance, refers to the spatial and geographical dimension: more specifically, it includes all the variables acting as 'resistance factor' that affect negatively the phenomena examined. This law of general gravity for social interaction can be expressed in the following formula notation:

$$F_{ij} = \frac{G\,(M_i^{\alpha}\,M_j^{\beta})}{D_{ij}^{\theta}} \tag{7}$$

Where F_{ij} is the flow created between i and j or, alternatively, the total volume of the interactions between i and j (that is the sum of the flow in both directions: from i to j and from j to i). It can represent, for example, the value of exports or migration. M_i and M_j are the relevant dimensions of the two subjects i and j (regions, countries, population, enterprises or others), while D_{ij} is the distance[32] between subjects i and j (usually measured from centre to centre or with an estimation of the resistance factor); α, β and θ[33] are parameters that adapt according to the type of social interaction taken into consideration (migration, tourism, foreign direct investment, et cetera). For the flow of people, it is more natural to measure M_i and M_j as populations. Note that we return to Newton's law equation (2) if $\propto = \beta = 1$ and $\theta = 2$.

According to the vector form of the law of gravity in physics, the property distinguishing gravitational models refers to the total flow from a source ($Origin_i$), which is the sum of that flow directed to all the possible destinations. Likewise, the total flow to a point ($Destination_j$) refers to the total flow that is directed from all origins towards that point. In formal terms (Philbrick 1973: 40-41):

$$Origin_i = F_{i1} + F_{i2} + F_{i3} + \cdots + F_{in} = \sum_{j=1}^{n} F_{ij} \tag{8}$$

$$Destination_j = F_{1j} + F_{2j+} + F_{3j} + \cdots + F_{nj} = \sum_{i=1}^{n} F_{ij} \tag{9}$$

$Origin_i$ is the total outward flow from i to all j. $Destination_j$ is the total inward flow terminating at a particular destination zone, j, from all origins, i. Gravity models are usually used to estimate the magnitude of the flows (F_{ij}) through the sum of $Origin_i$ and $Destination_j$, or to evaluate the total magnitude for a source or perceived at a point. According to Philbrick (1973)[34], models of the first type can be designated as gravity demand models, while the second as distribution models.

2.2 Pownall's Political Model of Gravity

The British colonial official and politician Thomas Pownall (1764) applied, albeit in a purely theoretical way, the Newtonian concept of gravity and attraction

[31] The size can be measured as gross domestic product, population, product offering, or other variables.

[32] In most of recent studies the distance is not only geographical, but it considers also different elements (morphology, communication infrastructures, cultural factors, etc.).

[33] The square of the distance, which in physics specifies a precise empirical relationship between different measurement systems (for masses, distances and forces), is left apart in some studies in the fields of social sciences. Unlike physics, in these cases the relationship is more theoretical than empirical, since the latter provides index that cannot be measured directly in nature. It is worth noting these two different approaches in the next paragraphs. For example, for Stewart (1941) we will see that the square of distance is not superficial but substantial for his studies in the social sciences. As in physics, in fact, he makes a distinction between his formulas of 'demographic energy' and 'demographic gravitation'.

[34] Philbrick makes this distinction for the 'Transportation Gravity Models' which can however be taken up in a more general sense (Ibid.)

to his theories of empire in order to describe a system of economic and political influence between states and across different social classes.[35] Historically, these studies represent a stand-alone theory, which are remarkable for the breadth of their theoretical scope. Pownall's work anticipates in a universal way many of the successive applications of gravity without, however, limiting its use to a single field of study.

According to the British politician, political power gravitated around property, so that every government in history has always reflected the wealthiest part of the population of that specific time. The gravity of property was exerted between different classes and between different regions in space. The latter was perceived as the area where different gravitational fields in the property interacted with each other. Thus, for example, the Colonies of Great Britain gravitated around the mother country for its economic power.

A change in the structure of ownership between different classes or between different regions would automatically imply a shift in the political system. The gravity centre of power was neither fixed nor permanent but rather mobile, and it followed the economic power. This, at Pownall's time, found its greatest expression in trade. The relationships were based on the interconnection of productive activities, among which work was the gravitational force that held together and regulated society.[36] Against this backdrop, the official had predicted that over time, because of the property's law of natural gravity, the centre of gravity would have shifted from Britain to the colonies of North America, or more generally from European space to the Atlantic areas, which thanks to capital accumulation would have soon gained commercial and economic primacy.

2.3 Application of Gravity to People's Movements

The first systematic and scientific applications of gravity to the social sciences trace back to the 19th century.[37] On top of all social interactions, the study of gravity for immigrant flows was the first to be conceptualized.[38] Metaphorically, as already mentioned, these studies transpose gravity's universal law, the product of the masses and the inverse of the square of distance to human choices (Sen and Smith 1955). With regard to migration and displacement, the gravity model is used in urban geography to estimate traffic flows, migration between two areas and the attraction of people to a centre or to multiple centres. The idea is that more populous and less distant centres have a greater flow of movement between them. Migratory phenomena might or might not be linked to free movement areas of people and migration laws. On the one hand, they are connected since these movement areas and laws often tend to play a decisive role for migratory flows. On the other hand, since migration cannot always be regulated and controlled, movement areas and migration laws are not completely effective. Many elements such as politics, language, culture, taxation and more have an impact on the distance perceived by migrants or even on a country's attractiveness. Such factors are in turn influenced by agreements on free trade and, more generally, on movements of people.[39]

First, Carey (1858: 42) applied Newton's universal law of gravity to "railway traffic and migration" by tracking a tendency for the individual to "gravitate to his fellow man". According to the scholar, the individual, as a component of society ('molecule of society'), is subject to a force of social attraction to other social agents, just as matter in physics is subject to a force of gravitational attraction with other matter. Ravenstein (1885) described this concept in the migration law, according to which a 'populated centre' attracts migrants from other populated centres in relation to the size of the population and inversely related to distance between centres. The mathematical function is as follows:

$$M_{ji} = f\left(\frac{P_i}{D_{ij}}\right) \qquad (10)$$

where M_{ji} is the migration flow from centre j to centre i[40], $f(P_i)$ is a function of population size i and

[35] One of the main objectives was to analyse the geopolitical balances of the British Empire.

[36] The interested reader will be able to deepen in: Matilde Cazzola, "Space as gravitational field: the empire and the Atlantic in the political thought of Thomas Pownall." Global Intellectual History 2(2018): 178-201.

[37] The idea that human behavior varies in a non-linear way, but with the square with respect to a given variable, has been "powerful and durable" over time (Anderson 2004: 334). In the social sciences, we can find the use of the square in different works. For example, Malthus predicts continuous problems of sustainability of the geometric growth of the human population with respect to the arithmetic growth of necessities. (Malthus 1798). In 1890, the statistician and sociologist Francis Galton, father of the regression, realized that for the mathematical use of the square of the deviation "there seems to be a wide field for the application of these methods to social problems" (Galton 1890). However, especially since the use of the law of gravity in the social sciences, the idea of a squared exponent of distance has not always been used. It was rather maintained a gravitational structure that takes into account the attraction force (in general called 'magnetism') of a variable considered and a more generic use of friction or resistance of the distance calibrated on the specific phenomenon considered (Sen and Smith 1955). However, this is not true for all scholars, as we will see for example in Stewart's (1941) model, which gives theoretical importance to the square of distance.

[38] The interested reader can also see the studies by Helliwell in 1997 and Portes and Rey in 2005.

[39] For these reasons, I considered trade agreements as the third macroeconomic lever of the economic power and the agreements on people's movements as the fourth invisible macroeconomic lever. The latter is invisible because these agreements are more difficult to evaluate economically.

[40] This should be clear considering that in the formula it is the population of the country i that attracts the migratory flows.

D_{ij} is the distance between the two population centres.

Contrary to Carey's argument, Young (1924), whose migration formula is based on the gravity law of physics, states that human behaviour "does not lend itself to exact mathematical formulation". This approach is similar to the one used by Ravenstein:

$$M = k\left(\frac{F}{D^2}\right) \tag{11}$$

where M is the estimate of migration between two communities, k is a constant, F is the intensity of attraction and D is the distance between the two communities. Therefore, mutual migration will be directly proportional to the intensity of attraction and inversely proportional to the square of the distance between the communities.

In subsequent years, Reilly (1929) extended the concept of social gravitation to consumers and cities.

The author describes how the city's force of attraction exerted on consumers in the surroundings is directly proportional to the size of the population, and inversely proportional to the square of the distance between the two.[41] Therefore, ceteris paribus, consumers will be more attracted by larger cities rather than by smaller ones. Nonetheless, in accordance with Newton's law, the level of attractiveness would be neutralized at the specific 'breaking point', where consumers will be indifferent to either of the two competing cities, as demonstrated in the following formula (Reilly 1931):

$$\frac{P_i}{D_i^2} = \frac{P_j}{D_j^2} \tag{12}$$

where P_i and P_j are, respectively, the size of the centre i and the centre j, D_i is the distance of the indifference point from the centre i and D_j is its distance from the centre j.

The astrophysicist John Q. Stewart (1941) applied the laws of physics to the study of social sciences, thereby creating the theoretical basis for subsequent studies in social physics. Through his studies of the empirical regularities associated with distance in the social sciences, the scholar found that the law of gravity could also be used to describe demographic phenomena.

To that end, the author adopts the concept of demographic gravitation, according to which a large number of people (for example the population of a big city) acts as force of attraction for other individuals who eventually choose to migrate there. The formula is as follows:

$$F_{ij} = k\left(\frac{P_i P_j}{D_{ij}^2}\right) \tag{13}$$

where F_{ij} is the interaction force between the demographic centres i and j, k is a constant[42], P_i is the population of the area i, P_j is the population of area j, and D_{ij} is the distance between i and j.[43] More specifically, Stewart proposed to adopt different values for the population by nationality, considering the 'molecular weight' of each population's member. With the aim of standardizing the measures, the author chose the molecular mass of the American average as a unit. The latter, for example, will be distinguished from an Australian aborigine, whose molecular weight will presumably be less than one.

The theory further evolved with the concept of demographic energy and potential of population by Stewart (1948). The author measured on the one hand the mutual energy between the populations within the gravitational field and on the other hand the intensity of the magnetism of a demographic centre. The astrophysicist's hypothesis was that demographic energy can be interpreted as the number of human relations per unit of time.[44] Stewart made assumptions of applicability while leaving space for further hypotheses and possible developments in the future.

The content and method of his approach was based on the suggestive interdisciplinary formulas rooted in physics, that in the author' s view, can be successfully applied to the social sciences. He was rather convinced that human phenomena could be studied with the same rigour and through the same laws of physics that regulate nature. Hence, the scholar developed hypotheses of application, which would be intrinsically coherent with the rules of physics. Since Stewart's formulas are entrenched in physics, this approach should never be abandoned. Indeed, the future application of such equations to other socio- and econophysics fields of research should maintain its physical foundation to avoid the

[41] The distance considered by this law is a straight line: obstacles such as mountains, rivers, deserts, etc. are not taken into account.

[42] Stewart postpones the determination of G to the future, with the suggestion of simplifying the units of measurement of population and distance to reduce it to unity.

[43] Like Reilley, Stewart considers straight line distances, while providing several examples where distance should be otherwise interpreted.

[44] Stewart (1948: 56) compares human relations to an "impulse which happens". Moreover, the author adds that the accumulating total of such happenings is the integral of energy with respect to the time, a concept that is similar to 'action' in physics.

risk of conceptual mistakes.[45] Starting from the concept of demographic gravitation, he advanced in demographic analysis following the same laws that are valid in physics. In addition, he used the concept of demographic gradient which simply measures the concentration of people per square mile. In formal terms[46]:

$$E_{ij} = k\left(\frac{P_i P_j}{D_{ij}}\right) \quad (14)$$

$$V_{ij} = K\left(\frac{P_i}{D_{ij}}\right)$$

$$gradient = K\left(\frac{P_i}{m^2}\right)$$

being E_{ij} the demographic energy of the two populations, V_{ij} is the population's potential for attraction to centre i, K is a constant, P_i is the population[47], D_{ij} is the distance, and m^2 is a square mile.[48]

Considering more than two population centres in space, the total potential will be equal to the sum of the bilateral potentials. Stewart (1948) developed a simplified map where the population is distributed on a flat and continuous surface[49] and the potential at any point produced by the entire population was equal to[50]:

$$V_{at\ any\ point} = \int \frac{1}{D} D_P dS \quad (15)$$

D_P corresponding to the density population in the area dS, and D the distance from that area to any point taken into consideration. This represents the total interaction between a subject at one point and the remaining population of all other areas. Taking into consideration the vector property already mentioned, this can also be expressed at point i as (Philbrick 1973: 42):

$$V_i = k\sum_{j=1}^{n} \frac{P_j}{D_{ij}} \quad (16)$$

where V_i is the population's potential for attraction to centre i, K is a constant, P_i is the population and D_{ij} is the distance.

The astrophysicist showed in several studies how the result of his formula could be used to draw a map of the surface by means of the device of 'contours of equipotential'. The latter reminds of a 'synoptic weather chart',[51]as demonstrated in Figure (1) below produced by Stewart. This is also quite evident from the fact that formula (15) is analogous to and derives from the formulas of gravitational fields (6) in physics.[52] The graphic representation assumes the characteristics of a gravitational or magnetic field with its contours of equipotential.

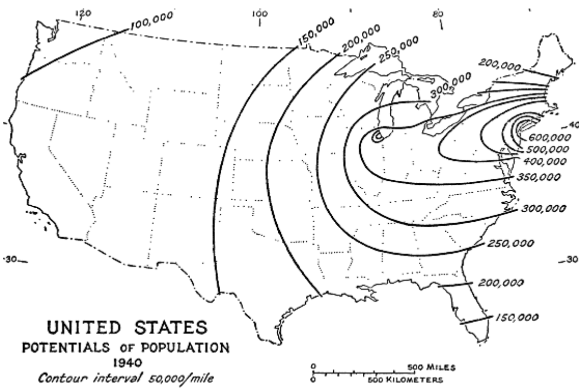

Fig. 1 Contours of equal population potential for the United States, 1940, Source: Stewart (1948)

The potential at any point produced by the entire population can be understood in metaphorical terms as a measure of how close people are to that point. The contribution offered by each person decreases with age.

In addition, the total demographic energy of the whole country can be obtained by combining the formulas (5) and (6) in physics, after several steps it is obtained that:

$$2E_{ij} = V_{ij}P_j + V_{ji}P_i \ \rightarrow \sum E = \sum P_{state} V_{state} \quad (17)$$

[45] When considering this model, the laws of physics are still valid and they are transposed upon the social phenomenon under analysis. Therefore, any new development or application of socio-physics should comply with such underlying rules.

[46] As seen in physics, in the formula of gravitational potential energy and in that of gravitational potential there is a negative sign because energy is identified with the mechanical work necessary to separate the two particles at an infinite distance, which here is simplified by the scholar as not conceptually significant for the population (Stewart, Ibid., 34-35).

[47] Anderson (1956) considered a stronger distance impact for small centres than for large centres.

[48] Carrothers (1955) suggested that the resistance effect increases with distance but less than proportionally.

[49] Indeed, the method of gravity's representation is generally complex due to its three-dimensional dimension.

[50] Stewart derived this formula from physics: $V = \int \frac{1}{r} DdS$. It considers the potential at any point in a plane and with the presence of multiple masses. In the formula D is the surface density of mass over the infinitesimal element of the area dS, r is the distance from that element to the point under consideration, and the integration is extended to all areas of the plane where D is different from zero.

[51] Also known as 'weather map', it describes the evolving meteorological situation, including pressure and wind, by means of circular lines. Hence the similarities with Stewart's map.

[52] However, it considers the sum of several gravitational potentials through the use of summation.

where P_{state} is the population of a country, and V_{state} is its average potential caused by another country.

Stewart applied these formulas to demonstrate a direct relationship between the demographic energy and other variables, such as the real estate market, the rental market, the value of income, the exchange of letters. This phenomenon acts in space as a kind of gravitational field. For example, starting from the suggestion that rents and the value of land tended to increase as population density increased moving from rural areas to cities, his empirical analysis demonstrates that the demographic energy was proportional to the economic wealth in the US.

Similarly, to Stewart's[53] first formulation, Zipf (1946) described a theory of the movement of people between two communities separated by a certain distance through the formula:

$$F_{ij} = P_i \frac{P_j}{D_{ij}} \tag{18}$$

where F_{ij} is the flow of people between two communities, whose populations are indicated with P_i and P_j respectively, while D_{ij} represents the communities' distance. The scholar also applied his model to the study of the interchange of telephone calls, bus passenger movements, newspaper circulation[54], and other phenomena (Zipf 1949).

In subsequent years, scholars used different weights for the population. By studying New Haven traffic, Rice and Gallagher (1948) adjusted the weights considering the wealth of populations.[55] Furthermore, they introduced the concept of directionally-zoned populations to take into account the "distance to the railroad station" (or to a terminal area), which influenced the traveller's choice of origin city. Stewart (1948) noted that some cities had greater influence, thus assigning different molecular weights to populations of different regions. Dodd (1950) proposed the use of different molecular weights according to sex, age, occupation, income, education and religion. I suggest that these different formulations in Philbrick (1973: 42-43) can be resumed in the formula:

$$F_{ij} = K \left(\frac{(W_i P_i)(W_j P_j)}{D_{ij}^2} \right) \tag{19}$$

where P_i and P_j are the populations (Rice and Gallagher considered the directionally-zoned population of the stations) and W_j and W_i are the weights of the two respective populations (for Rice and Gallagher a wealth factor, for Stewart and Dodd the molecular weights to be estimated).

After a few years, the engineer Alan Voorhees (1956) wrote "A General Theory of Traffic Movement", where he applied gravity to trip distribution.[56] His theory, which represented the first application of the gravitational principle to traffic analysis in urban areas, is used to estimate the number of trips between two zones. According to the scholar, each journey involves four fundamental decisions, often taken simultaneously: namely, the choice of travelling (opportunity cost of distance); the choice of a destination (attractiveness); the choice of means of transport; the choice of route (role of technology and information).

With respect to gravity formulas already seen, the number of trips is inversely proportional to distance, while the size variables considered for the two locations, such as population or level of development, are proportional to the number of trips.[57]

$$T_{ij} = K \frac{P_i P_j}{d_{ij}^x} \tag{20}$$

Drawing on such formulas, Voorhees developed a distribution model, where the estimated number of trips from the area of origin is divided by the destination area. This estimation is proportional to the attraction exerted by each area ('attractiveness of the soil') and is inversely related to the necessary travel time. Its formula is as follows (Philbrick 1973: ibid.):

$$T_{ij} = \frac{O_i \frac{S_j}{d_{ij}^x}}{\sum_{j=1}^{n} \frac{S_j}{d_{ij}^x}} \tag{21}$$

where T_{ij} is the trips between zone i and zone j, O_i is the number of trips generated in zone i, S_j is the attraction force of land j, d_{ij}^x is the travel time between i and j, x is an exponent determined by

53 Philbrick (1973: 42) points out that although Stewart's and Zipf's formulae are basically the same, Zipf's relationship differs because it raises the entire $\frac{P_i P_j}{D_{ij}}$ to a power.

54 He analyzes several newspapers including the Chicago Tribune and The New York Times.

55 Rice and Gallagher are the first to apply an adjustment by a wealth factor to the population, for example by making use of per capita income.

56 Voorhees was working at the time to predict traffic patterns in Baltimore.

57 Recent studies reject the idea that size variables (masses) are determined exogenously to the system: economic and demographic totals are usually exogenously given and not unknown variables which need to be determined. These models include attractiveness, which is sensitive to the transport network (Philbrick 1973: 45).

observation,[58] n is the total number of trips including those directed to j.[59]

Eventually, Hansen (1959) was inspired by Stewart and Reilly's ideas when defining the accessibility index (AI)[60], a spatial method for estimating the development of residential land in an urban area. The aggregate growth for a region was calculated by weighted proportionality. In formal terms:

$$AI_i = \sum_{j=1}^{n} \frac{S_j}{D_{ij}^x} \tag{22}$$

$$G_i = G_T \frac{V_i\, F(AI_i)}{\sum V_i\, F(AI_i)}$$

where AI_i is the accessibility index for zone i, S_j is a measure of the activity's size located at zone j (for instance number of jobs), D_{ij} is the separation between i and j with D_{ii} having a numerical value, x is an exponent expressing the effect of separation, G_i is the residential growth in zone i, G_T is the total regional residential growth, V_i is the vacant developable land in zone i, $F(AI_i)$ is a function of the accessibility of zone i.

2.4 Applications of Gravity to Economic Flows

The application of gravity in the study of migration[61], demography and trips comes along with its application to the study of international trade.[62] The economic model is directly inspired by Stewart's model and reproduces its logic and dynamics, but it transposes gravity to a flow of commerce and no longer to a flow of people. The application to the study of international trade traces back to the work of Walter Isard (1954), professor of regional sciences at the MIT University in the United States, but it became known in Tinbergen's (1962) standard bilateral form, which was directly related to Newton's Law of Universal Gravitation. Subsequently, it was re-explored by several economists. Isard was inspired by Stewart's model and introduced the concept of income potential in international trade as in formula (6). Thus, the author applies a gravitational field equation in physics to the study of international trade. In his hypothesis, geographical proximity favours trade due to several variables,[63] such as low transportation costs, cultural and institutional similarities between regions, etc. He stated that a gravity relationship should arise in any model that studies distance as a factor contributing directly to increased costs. Moreover, states' trade is proportional to their respective economic size, which is measured[64] through their Gross Domestic Products (GDPs) or Gross National Products (GNPs). The larger the GDP, the larger the national economy, the more a national population can import or export. Its formula in international economics is as follows (Isard 1954: 308):

$${}_i^1V = \sum_{j=1}^{n} {}_i^1V_j = \sum_{j=1}^{n} k\, \frac{Y_j}{d_{ij}^a} \tag{23}$$

where ${}_i^1V$ is the income potential produced by all countries upon state i and ${}_i^1V_j$ is the income potential produced by state j upon state i, Y_j is the income of state or region j, d_{ij} is the average effective distance between states i and j,[65] k is a constant comparable to the gravitational constant, a is an exponent determined by observation, which is lower than the squared exponent by Stewart.[66]

Later, Tinbergen (1962) developed the gravity equation commonly applied to subsequent studies of international economics, which was intuitive and derived directly from Newton's equation (1). The author was a pioneer of the empirical application of gravity to trade in his work "Shaping the World Economy". In this way, he created a simple bilateral

58 X indicates the distance resistance rate modelled on the number of trips when the origin and destination are the furthest apart. As the exponent increases, the level of attractiveness of the most distant destinations from the origin decreases. The ratio between S_j/d_{ij}^x represents the perceived utility of a person located at point i with respect to the destination at point j.

59 The division by the summation that considers the total number of trips (n). It simply divides the utility of j by the sum of the utility of all destinations including j (Voorhess 1956).

60 Hansen (1959) was the first to introduce the concept of accessibility as a potential of opportunities for interaction. This index, conceptually linked to Newton's law of gravity and by definition to localization theories, has been a great success in urban geography and spatial modelling over the last 60 years, as it considers both the type of activity of people and the connections of the means of transport.

61 More recently, Anderson (2011) has developed a gravity model for migration through discrete choice techniques and Ahlfeldt et al. (2012), drawing on the contribution of Eaton and Kortum (2002), have built model of commuting gravity.

62 Among the applications of gravity to the social sciences, the field of trade has been the most fortunate. The model was considered by Deardorff (1998) as "the workhorse for International Economics studies" for its predictive power in estimating exchange flows (Gopinath et al. 2014)

63 The income potential varies inversely to the distance. Between a group of states, the one more similar in culture and technology to nation i, will have a greater income potential on i.

64 In 1954 Harris (1954) and Isard and Freutel generalized the concept of mass in the gravitational model with other variables and used transport costs rather than distance costs.

65 Note that distance as applied in trade is part of the 'resistance factor' which includes all the variables acting as trade barriers, as a kind of 'wedge' tax imposing costs and influencing choices. Moreover, according to Isard, distance should cover the relative movement of goods on diverse means of transport.

66 The attentive reader will have noticed that, compared to Stewart, Isard loses the squared exponent. Therefore, although Isard's model also follows sociophysics, it does so to a lesser extent than the astrophysicist.

model for the analysis[67] of trade between pairs of countries.[68] The formula is the following:

$$F_{ij} = A \frac{Y_i Y_J}{D_{ij}} \tag{24}$$

where F_{ij} is the value of trade between country i and country j, A is a constant comparable to the gravitational constant, Y_i is the GDP of country i, Y_J is the GDP of country j, D_{ij} is the distance between the two countries. [69]

Gravity models have also been used similarly and extensively in economics to evaluate FDI flows. Building on the model by Tinbergen (1962), which is suitable for all types of international trade flows, the pioneers of the application of gravity to such cross-border transactions seem to be Eaton and Tamura (1994), Graham (1994) and Frankel et al. (1996).[70] Indeed, governments increasingly perceive FDI as a driver for development[71]. Therefore, it plays a pivotal role in development policies and in negotiations on international trade agreements. Intuitively, the membership in trade agreements, which are set to decrease barriers, should help to increase investments. In addition, the existence of an official language, a common currency or a common legal system among countries should increase the opportunities for investment. In the case of FDIs, the formulation of the model is the following:

$$FDI_{ij} = A \frac{y_i y_j}{D_{ij}} \tag{25}$$

with FDI_{ij} representing the FDI flow between two countries i and j; A is a constant; y_i is the GDP or GNP of country i; y_j is GDP or GNP of country j; and, finally, D_{ij} is the distance between countries i and j.

Afterwards, the economist[72] and geographer David Huff (1963) re-applied[73] gravity to consumers' behaviour with the aim of forecasting market share and retail attraction. In a set of alternatives, the attractiveness of each retail for a subject is inversely proportional to the perceived utility of any other alternative retail. For example, among competing stores, space is conceptualized as the area of probability within which a customer could decide to shop at a given store. This probability is calculated through the use of distance and attractiveness variables and taking into account the competition. In Huff's standard model there is no differentiation of goods, no feedback between consumers, no alternative goods. The attractiveness of the retail outlet at location j on consumer demand at location i is defined as:

$$\frac{attractiveness\ j = size\ of\ retail\ outlet\ j}{D_{ij}^x} \tag{26}$$

The probability that this consumer demand[74] at location i is satisfied by the store j will be equal, as already seen in Reilly's equation (11), to the relative attractiveness of j compared to the attractiveness of other stores. As in the case of Voorhees's model, it is worth recalling that x is an exponent expressing the effect of distance decay. Ergo, total demand is distributed by multiplying expression (25). Huff therefore developed a distribution model of consumer demand in space drawing on Voorhees' model, as from formula (19). Similar to the latter, who chooses a spatial distribution of trips based on the attractiveness of the soil, Huff also considers a spatial distribution, which is related to the attractiveness of the commercial area. Huff's probability model is as follows:

$$P_{ij} = \frac{\frac{S_j}{d_{ij}^x}}{\sum_{j=1}^{n} \frac{S_j}{d_{ij}^x}} \tag{27}$$

where P_{ij} is the probability that a consumer at point i goes to the store located in j, S_j is a measure of store attractiveness (such as store size), d_{ij}^x is the travel time between i and j, x is an exponent

67 While Isard was interested in a complete theoretical approach and an all-inclusive analysis between economic geography and international trade, Tinbergen was more concerned about the empirical aspects.

68 The model predicts bilateral trade between pairs of states.

69 Tinbergen clarified that the economic size of the importing country plays a dual role, indicating both total demand and a sort of production diversity's degree. Tinbergen empirically found, however, that richer countries traded "less than normal", if we were to consider a linear relationship.

70 Other relevant studies were concluded by Frankel and Wei (1996), Brenton et al. (1999), Portes and Rey (2005), Helpman et al. (2008) and by Bergamo and Pizzi (2014). Further uses of the gravitational model in economics were those of Portes et al. (2001), Martin and Rey (2004), Portes and Rey (2005), who build a ravitational model of portfolio investment, and Okawa and van Wincoop (2012) who adapt the gravitational model to the world of international finance.

71 The increase in foreign direct investment flows began with globalisation and has continued over the years, but it was interrupted by a fall in flows due to the international financial crisis (during 2008).

72 David Huff, professor of Marketing and Geography, proposed a model that bridges geography and business. This model became popular over the years for its ease of use and its applicability to a wide range of problems. It has been extensively used by market analysts to locate stores, shopping malls, standard models for the industry and other types of retail establishments.

73 As, to some extent, already seen in Reilly (1929).

74 The demand is inelastic, that is, if there is only one store, it will be totally monopolized by it.

determined by observation, *n* is the total number of stores that also includes the store *j*. The larger the size of a store, the greater the level of attraction exerted on the consumer. The probability that customers choose that store, instead, decreases as distance increases.

After that, the economist Quandt (1965) created[75] a gravitational model of travel[76], namely the gravity demand model, linked to its socioeconomic variables with respect to the areas of origin and destination. According to the following formula (Philbrick 1973:46-47)[77]:

$$T_{ij} = G P_i^{\alpha_1} P_j^{\alpha_2} Y_i^{\alpha_3} Y_j^{\alpha_4} M_i^{\alpha_5} M_j^{\alpha_6} W_i^{\alpha_7} W_j^{\alpha_8} H_{ij}^{\alpha_9} C_{ij}^{\alpha_{10}} S_{ij}^{\alpha_{11}} \quad (28)$$

where T_{ij} is the trips from origin *i* to destination *j* by all modes, P_i and P_j are the populations of the two zones, Y_i and Y_j are their mean personal incomes, M_i and M_j are the population of the labour force employed in mining and manufacturing, $W_i$and W_j are the population of the labour force employed in white collar jobs, H_{ij} is the average travel time, C_{ij} is the average cost of travelling, S_{ij} is an index denoting the number of means of transportation available, G and α are parameters to be estimated.

2.5 Tobler's Law

A few years after the first applications of the Universal Gravitation Law to social sciences, Waldo Tobler (1970[78]), professor of Geography at the University of Michigan, formulated a general law for space science, which had a remarkable affinity with Newton's law and was named 'the first law of geography'. Indeed, Tobler was inspired by gravity models when formulating his theory. The author aimed at creating a more general spatial law through distance in geography, in such a way as to include both spatial interaction phenomena, among which gravitational models and more static phenomena.

This law, then used in social sciences, states that "everything is related to everything else, but near things are more related than distant things" (Tobler 1970). This mechanism, as already illustrated, is known as 'distance decay'. Accordingly, his theory was based simply on predictive spatial power by focusing on the general effect of distance friction, while leaving aside the cause of distribution. Moreover, it focused on '*everything*' instead of on measurable[79] masses as in gravity, leaving more space for the study of non-material concepts such as knowledge, culture, ideas, et cetera. Although it is more general, Tobler's law loses on the side of spatial interaction compared to standard gratIvational models. Indeed, the latter not only provided a static illustration of real distance decay phenomena in social science but were also rather dynamic. Gravity, as already observed, is inclusive of attraction logics and balance of forces between the various subjects considered. These modify the surrounding reality which cannot therefore be static.

The first law of geography, just like the gravity model[80], has been used over time to describe a series of spatial human interaction models (Rey 2015):

$$T_{ij} = \frac{P_i P_j}{D_{ij}^{\beta}} \quad (29)$$

where T_{ij} is the degree of spatial interaction (migration, trade flows, air travel, et cetera) between place *i* and *j*. T_{ij} is proportionate to the sizes of the population and inversely correlated to distance. P_i is the population at the origin and P_j at the destination.[81] D_{ij} is the distance between these locations.[82] The strength of decline in interaction with increasing separation is estimated by β.

[75] Later, it will be developed by the economist himself as well as by his colleagues and students. The interested reader is addressed to: Richard E. Quandt, "The collected essays of Richard E. Quandt", Edward Elgar Publishing 363(1992): 1-864.

[76] The work done by Quandt has been related to intercity travel (dynamic forecasting models) through several reformulations and adaptations of the original model.

[77] The keen reader will have noticed the following formula is similar to the general classical equation to the Cobb–Douglas production function.

[78] However, it is only since the 1990s that Tobler's Law has been rediscovered in literature and used in geographic and social research. (Daniel Z. Sui 2004).

[79] Today it is possible to describe the first law of geography through spatial autocorrelation or Moran's Index (1950). It is defined as a set of unit similar parameters in space. If they are spatially concentrated, there will be a positive spatial autocorrelation. Otherwise, there will be a negative spatial autocorrelation (or spatial heterogeneity). For example, the first law of geography was a conceptual basis of geostatistics, the branch that deals with evaluating spatial autocorrelation of data and checking whether observations made around nearby points have a greater correlation.

[80] In literature, often gravity model and first law of geography are considered a single model.

[81] In this formula, population is used as a variable for size. However, as mentioned, Tobler's model was general and applicable to several variables, such as telephone numbers, income, trade flow, marriage, etc.

[82] Tobler generally used Euclidean distance, but he was open to different variables. Therefore, distance is estimated in the literature using various geographical and commercial policy variables, such as bilateral distances, tariffs (dollars), the presence of the same or different currencies, the presence of regional agreements (RTAs), etc.

Tobler (1970) also tried to identify not only the flow of a spatial interaction but also the distances and positions of the locations, with flows as data values and distance as an unknown variable. Therefore, he was reversing his law by stating that "all things can be located on a map so that similar things are placed closer together than dissimilar things" (Waters 2018:4), thereby creating a deterministic gravitational model of both flows and distances. Thus, the previous formula turns into:

$$D_{ij} = \left(\frac{P_i P_j}{T_{ij}}\right)^{1/\beta} \tag{30}$$

In Tobler (1970), the geographer for the first time applied the reversal of gravity model to derive empirically the position and the diffusion of botanic species between the islands of New Zealand. He demonstrated that distance[83] and size alone can explain up to one-third[84] of the floristic biodiversity between the islands.

In the same vein, Tobler and Wineburg (1971:39-41) estimated the unknown positions of a series of Assyrian settlements between 1940 BC and 1740 BC for which commercial data were available.[85] Using the gravity model to calculate distances, mapping became easier with increasing estimates of distance pairs.[86] In their hypothesis, the more settlements traded with each other, the more similar they were in terms of location. They also assumed that the name occurrences of larger towns were more frequent with respect to smaller ones. [87] Overall, it is both the size of the cities and the distance between them that determine their interactions.

With respect to the applications deriving directly from gravity, Tobler's law, although strictly related to the former, adopts a more general approach focused on geometry. If compared with the eco-physical approach, which proposes the notions of attraction, magnetism and economic relationships between agents, Tobler's model is biased towards geography, distance decay and phenomena of concentration and dispersion.

2.6 Most Recent Applications

Since 2000 a series of spatial interaction's gravity models has been developed under the name of "two-step floating catchment area" (2SFCA) for the accessibility to some public and private services mainly related to health, such as those of GPs, cancer treatment facilities, access to healthy food retailers, et cetera. This method based on spatial decomposition was advanced by Radke and Mu (2000), who combined several related information in a single significant index that allowed for comparisons between services accessible in different locations. More specifically, these models can be divided into five research categories. While the first involves the original dichotomy form of distance decay, the second includes discrete multilevel or continuous shapes also considering density. The third category refers to the development of river basins and the fourth takes into account the competition between supply and demand. Lastly, the fifth considers the means of transport, which integrate into the model, for instance, commuter transport.[88]

The gravity model has also been applied to studies on digital networks. Firstly, and from a theoretical perspective, Batty (1997) introduced the concept of virtual geography, while Gorman and Malecki (2000) coined the term 'cyber-geography' to describe the Internet as a non-homogeneous system with different levels of diffusion across geographical areas. The gravitational metaphor has moved from the study of physical to virtual space. These models studied mutual relationships of the two areas, namely the effect of digitalization on physical distance (Reggiani et al. 2010; Tranos 2011), and the impact of the latter on digital space (Barabási et al. 2012; Newman 2003 and Watts 2004). The most recent applications of digitization by Tranos and Nijkamp (2013) highlighted that the spread of the internet is in turn conditioned by physical, technological, and institutional distance, following a model similar to Newton's Gravitational model.

Anderson et al. (2015) introduced a structural gravity model to analyse international trade in different services, sectors, countries and time. The authors paid greater attention not only to trade distances, but also to variables that are important for services, such as institutions, digital infrastructures, et cetera thereby creating a new methodology for deriving services' border barriers, which by their nature are difficult to obtain. According to the scholars, in fact, little is known about the cost of trade in services, a sector that is characterized by the lack of data. This is due to the absence of transport

[83] Despite using a geometric distance, Tobler was aware that this was a simplification of reality.

[84] Tobler did not believe that his model could explain all variations, but he thought that more sophisticated models could later be developed on this simpler basis. In that regard, it stated that: "Model-building is useful not only because it may allow prediction but also because it identifies areas for further research" (Claude Grasland 2010: 4).

[85] On the archaeological data found by Hrozny near the village of Kültepe in 1925.

[86] Tobler located the cities through a process of comparison between estimates of the inverse of the law of gravity and archaeological sites discovered in that region.

[87] The size of the cities was not known, neither was the value of the flows. Tobler estimated the size of cities from the frequency of quotations and flows from the common citation of names.

[88] The interested reader will be able to deepen in Luo and Wang (2003) and 2005, Luo and Qi (2009).

costs, opaque laws and difficult-to-measure trade's tariffs in professional and financial services compared to trade flows of goods. Anderson et al. (2015) concluded that the gravitation model fits well with the service sector. Over time, border barriers have generally decreased for larger countries with differences by sector, while staying fairly stable for smaller economies. In addition, the negative distance effect seems to be stronger for neighbouring areas and it is reduced more than proportionally over long distances. On the contrary, language effects are more influential for services trade than for goods trade.

More recently, Bonchek (2020) suggested the application of gravity to business economics and marketing. Indeed, once companies have built their brand identity, they produce a gravitational field that attracts customers into the orbit of their brand. [89] In the most famous companies, this field of attraction would be created through three steps: shared purpose, engagement platforms and collaborative partners. While in the first step the attention is not on the immediate purchase, but on sharing a purpose to attract future consumers, the second addresses engagement platforms keeping the customers in the orbit around the brand. The last step consists of creating partners that amplify the gravitational field of the brand by multiplying the value of services and strengthening corporate credibility. By following these phases, the company builds relationships with its consumer base through authenticity and social connections.

3. CONCLUSIONS

The socio-physical importance of the gravity model has been widely discussed, starting with the birth of the model in physics and its transposition to several domains, especially those related to human behaviours and social phenomena. To that end, attention was paid to the exposition of Newton's law and to the preservation of the physics approach as a method of analysis, while also introducing the concept of 'social-physics'. A great deal of consideration was given to the most common applications of the gravitational model, as for economic, migratory, and demographic fields. The review followed a chronological order of presentation that favours a better understanding of the development of the use of gravity in the social sciences over time. Furthermore, a division by topic was outlined.

In particular, considering all the models scrutinized, two large lines of study could be distinguished (with the exceptions of Pownall's model): Tobler's first law of geography, and the most recent models of gravity. Although they are linked to other models by the common application of gravity to social sciences, these lines of thought developed more independent theories. Following a chronological order, the first study focused on the flow of people declined in its specific models such as migration, demography attraction, trips, et cetera. Drawing on the latter, the second one studies economic flows (trade, investment, etc.), thereby shifting from the investigation of people's movement to the movement of things. The link between these two fields of study can be historically found in the seminal works by Stewart and Isard. Indeed, through the wide extension of physics to his social model, Stewart laid down the theoretical foundations for subsequent models. Isard, who was inspired by Stewart's demographic approach, developed his model of international trade. In light of the most recent models reviewed and partly due to the more general nature Tobler's first law of geography, a new line of study seems to emerge. The latter moves from the analysis of material things (either individuals, money or goods) to less tangible concepts, such as the offer of services, brand identity, et cetera.

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SOME ASPECTS OF THE SOLUTIONS OF ROSSLER SYSTEM AND CHAOS

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Abstract. *In this paper we have studied the three dimensional non-linear system of differential equations by taking in consideration the two dimensional phase space of the Rossler system. The characteristics of the system are studied in reference to periodic, quasi-periodic and chaotic behavior. The conclusions are supported by means of diagrams of the trajectories obtained in two and three dimensional spaces, strange attractors and bifurcation diagrams.*

Keywords: *Nonlinear systems, trajectories, phase spaces, bifurcation diagrams, strange attractors.*

1. INTRODUCTION

A wide range of physical phenomena where there is a change in one quantity that occurs due to a change in one or more quantities can be mathematically modeled in terms of differential equations. Differential equations can be used to describe the motions of objects like satellites, water molecules in a stream, waves on strings and surfaces, etc. In this section we will take a review of some basic terminology associated with a system of differential equations.

1.1 System of Differential Equations [9]

Let $x_1(t), x_2(t), \dots, x_n$(t) be differentiable functions of a variable t defined on an interval I. Let $f_1, f_2, \dots, f_n$ be functions of $x_1, x_2, \dots, x_n$ and t. Then the set of n differential equations $X' = F(X,t)$, where $X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$,

$X' = \begin{bmatrix} x_1' \\ x_2' \\ \vdots \\ x_n' \end{bmatrix}$ and $F = (f_1, f_2, \dots, f_n)$ is called as a system

of differential equations. The system where F can depend on the independent variable t is called as a non-autonomous system [2] otherwise an autonomous system which can be written as $X' = F(X)$. The system $X' = F(X,t)$ is said to be linear or non-linear according as the function F is linear or non-linear. Every solution of the system represents a curve in R^n which is called a *trajectory*.[3] Trajectories help us to study the qualitative behavior of a system. The function F is also called as a *vector field* [4]. The vector field always dictates the velocity vector X' for each X. A picture which shows all qualitatively different trajectories of the system is called as a *phase portrait*. [4]

A linear system of differential equations can be expressed as $X' = A.X$, where A is an $n \times n$ matrix. A theorem concerning the uniqueness of the solution of a linear system is stated as follows.

1.2 Theorem (The Fundamental Existence - Uniqueness Theorem) [6]

Let E be an open subset of R^ncontaining X_0 and assume that $F \in C^1(E)$. Then there exists an $a > 0$ such that the initial value problem $X' = F(X)$, $X(0) = X_0$ has a unique solution $X(t)$ on the interval $[-a, a]$.

The fundamental existence and uniqueness theorem can be also stated as follows. A function $F: R^n \to R^n$ is said to satisfy Lipschitz condition on a domain $D \subset R^n$ if there exists a constant α such that

$$||F(X_1 - X_2)|| \leq \alpha ||X_1 - X_2|| \text{ for all } X_1, X_2 \in D$$

If F is continuously Lipschitz then the autonomous system $X' = F(X)$ has a unique solution for an initial point $X(0) = X_0$ in the domain D.

A *critical point* [10] (equilibrium point, fixed point, stationary point) X_0 is a point that satisfies the equation $X' = F(X) = 0$. If a solution starts at this point, it remains there forever.

A critical point X_0 is called *stable* critical point of the differential equation $X' = F(X)$ if given $\epsilon > 0$, there is a $\delta > 0$, such that for all $t \geq t_0$, $|| X(t) - X_0(t) < \epsilon$ whenever $|| X(t_0) - X_0 (t_0) || < \delta$, where $X(t)$ is a solution of $X' = F(X)$. A critical point that is not stable is called an *unstable* critical point.

2. THREE DIMENSTIONAL SYSTEMS [19]

A three-dimensional linear autonomous system has the form $X' = AX$

where

$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, X' = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}, \ A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}.$

Here the coefficients a_{ij} are constants. It is clear that this system has at least one critical point $O = (0,0,0)$. The stable and unstable *manifolds*[5] of the critical point O are respectively defined by $E_S(O) = \{X: \Lambda^+(X) = O\}$ and $E_U(O) = \{X: \Lambda^-(X) = O\}$ where $\Lambda^+(X)$ and $\Lambda^-(X)$ are the *positive limit set* and *negative limit set* of the point X respectively.

2.1 Some Examples of Three Dimensional Linear Systems

We consider some examples of the three dimensional linear systems and discuss the nature of their fixed points and analyze the nature of the solutions.

Example 1. Consider the linear autonomous system of equations given by

$$x' = x + 2z, \quad y' = y - 3z, \quad z' = 2y + z.$$

Here, the system can be expressed as the matrix equation

$X' = AX$, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, X' = \begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \\ \frac{dz}{dt} \end{bmatrix}, \ A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 2 & 1 \end{bmatrix}$

Solving the equations

$$x' = 0, \quad y' = 0, \quad z' = 0,$$

we get the critical point $O = (0,0,0)$. The three solution curves are given by the equations

$$x(t) = \frac{-e^t}{3}\left[-3C_1 + \sqrt{6}C_2 \cos\sqrt{6}\,t - \sqrt{6}C_3 \sin\sqrt{6}\,t\right],$$

$$y(t) = \frac{\sqrt{6}e^t}{2}\left[C_2 \cos\sqrt{6}\,t - C_3 \sin\sqrt{6}\,t\right],$$

$$z(t) = e^t\left[-3C_1 + \sqrt{6}C_2 \cos\sqrt{6}\,t - \sqrt{6}C_3 \sin\sqrt{6}\,t\right],$$

where C_1, C_2, C_3 are arbitrary constants.

It can be easily verified that the characteristic polynomial of the matrix A is given by

$f(x) = x^3 - 3x^2 + 9x - 7.$

Solving $f(x) = 0$, we get the eigenvalues and the corresponding eigenvectors given by $\lambda_1 = 1,\ v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \lambda_2 = 1 + \sqrt{6}i,\ v_2 = \begin{bmatrix} -\frac{\sqrt{6}i}{3} \\ \frac{\sqrt{6}i}{2} \\ 0 \end{bmatrix},\ \lambda_3 = 1 - \sqrt{6}i,\ v_3 = \begin{bmatrix} \frac{\sqrt{6}i}{3} \\ -\frac{\sqrt{6}i}{2} \\ 0 \end{bmatrix}$

Since the real part of the imaginary eigenvalues is positive, the fixed point $O = (0,0,0)$ is an unstable fixed point and the curve is a rotating curve.

3. THE ROSSLER SYSTEM AND ITS SOLUTION CURVES

In this section, we will consider the three dimensional non-linear system known as the Rossler system which is given by

$$x' = -y - z, \ \ y' = x + \alpha y, \ \ z' = \beta + xz - \gamma z$$

where α, β and γ are parameters. We will investigate the trajectories of the system by fixing $\alpha = \beta = 0.2$ and varying the parameter γ. But first, we will have a brief discussion about the *limit cycle* [18] of a system. Limit cycles are the closed trajectories which are isolated i. e. the neighboring trajectories are not closed. A limit cycle is called as stable or attracting of the neighboring trajectories tend towards the limit cycle, otherwise the limit cycle is known as unstable. For two dimensional systems, Poincare - Bendixson theorem [10] states that if a trajectory lies within a closed bounded region in the plane containing no fixed points, then the trajectory approaches to a limit cycle and no strange behavior is observed in the system. However, in three dimensional or higher dimensional systems, the Poincare - Bendixson theorem is not applicable and the trajectories may be trapped and just keep moving inside a bounded region without approaching towards a fixed point or a limit cycle.

3.1 Nature of Solution Curves for Different Values of the Parameter γ

As mentioned earlier, in order to analyze the nature of the solution curves, we will keep the values of the parameters $\alpha = \beta = 0.2$ fixed and change the parameter γ and study the nature of solution curves. First, we choose $\gamma = 1$. In this case, we can observe a limit cycle of period one. The projections of the trajectories showing limit cycles of period one on the *xy*-plane, *xz*-plane and *yz*-plane and the periodic behavior of the solution curves obtained using the MATLAB programming are as shown in the figure 1.

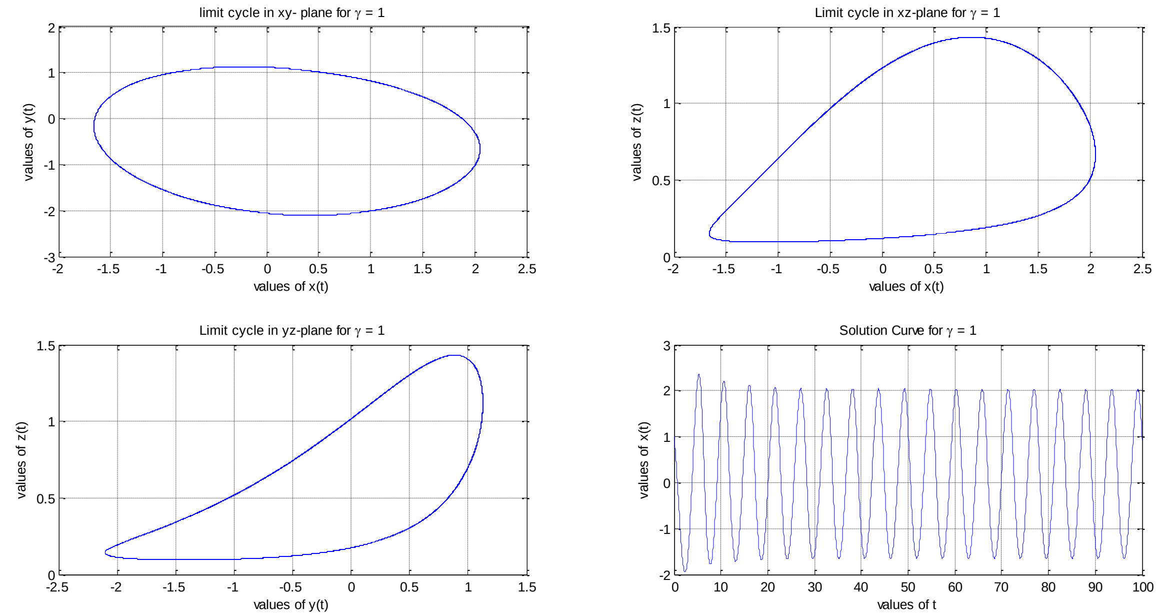

Fig. 1: Limit cycles of period one and solution curve

3.2 Period Doubling Cascade and Bifurcation Diagrams

When γ takes the value 3.4, the period one limit cycle is transformed into a limit cycle of period two. Hence in between $\gamma = 1$ and $\gamma = 3.4$, a period doubling bifurcation of limit cycles must have occurred. Such kind of bifurcations can happen only in three or higher dimensional spaces. The phase space showing the limit cycles in two dimensional spaces and the periodic solution are as shown in the figure 2.

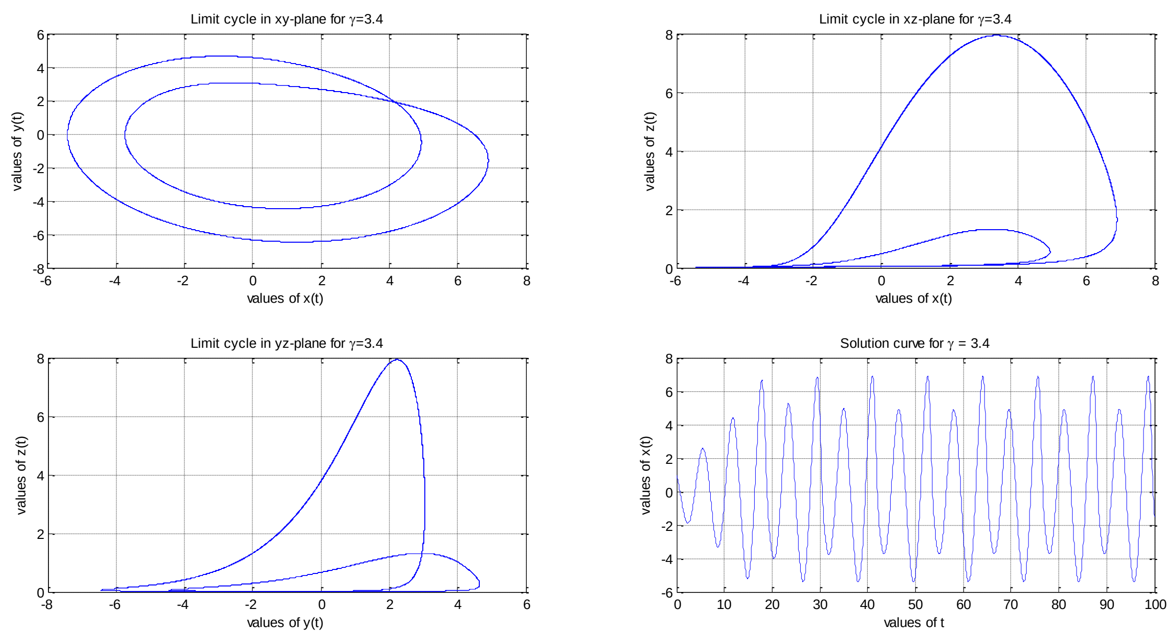

Fig. 2: Limit cycles of period two and solution curve

In the fourth subfigure of the figure 2, we can observe two distinct amplitudes. For the value $\gamma = 4$, a period four limit cycle is seen. This kind of period doubling phenomenon is observed infinitely many times as we go on increasing the values of the parameter γ. However, there is no exact method to find out the accurate values of the parameter where these bifurcations take place. The bifurcation diagram for the three parameters keeping any two of them constants are constructed using MATLAB and are as shown in figures 3, 4 and 5. The figure 5 showing the bifurcation diagram for the parameter γ can give us a clue about the approximate values of the parameter where bifurcations happen. It can be observed that the bifurcation diagrams are analogous the bifurcation diagram of the logistic family as shown in the figure 6.

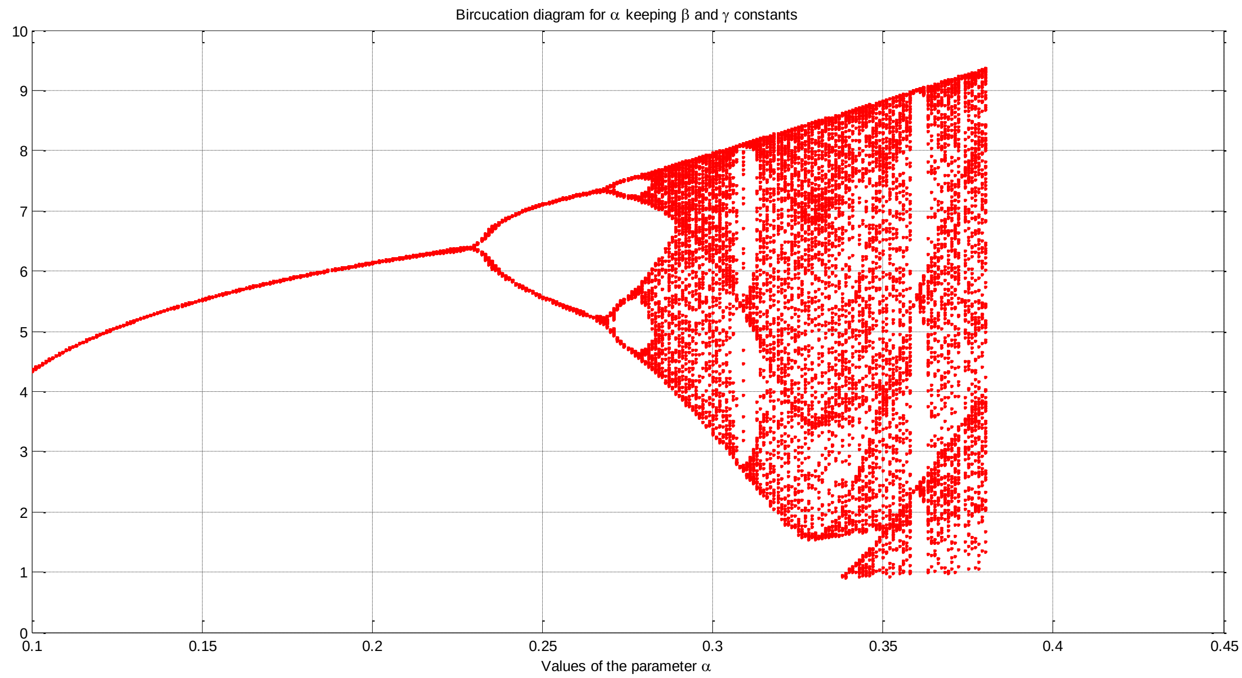

Fig. 3: Bifurcation diagram for α

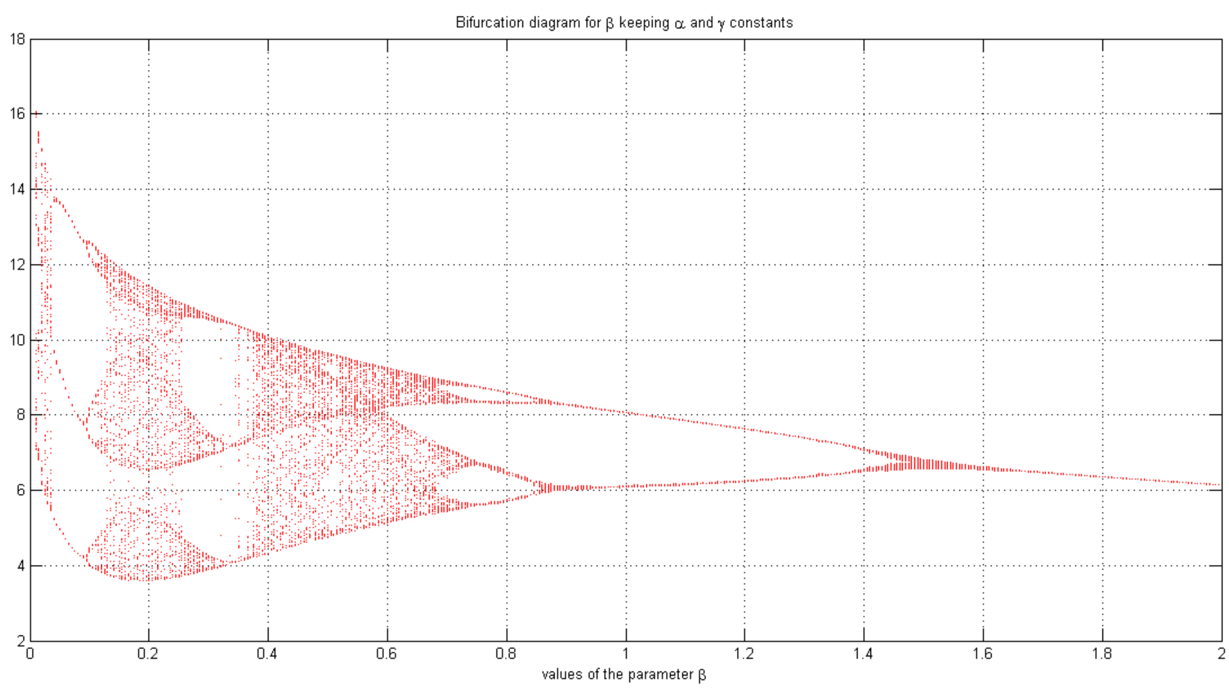

Fig. 4: Bifurcation diagram for β

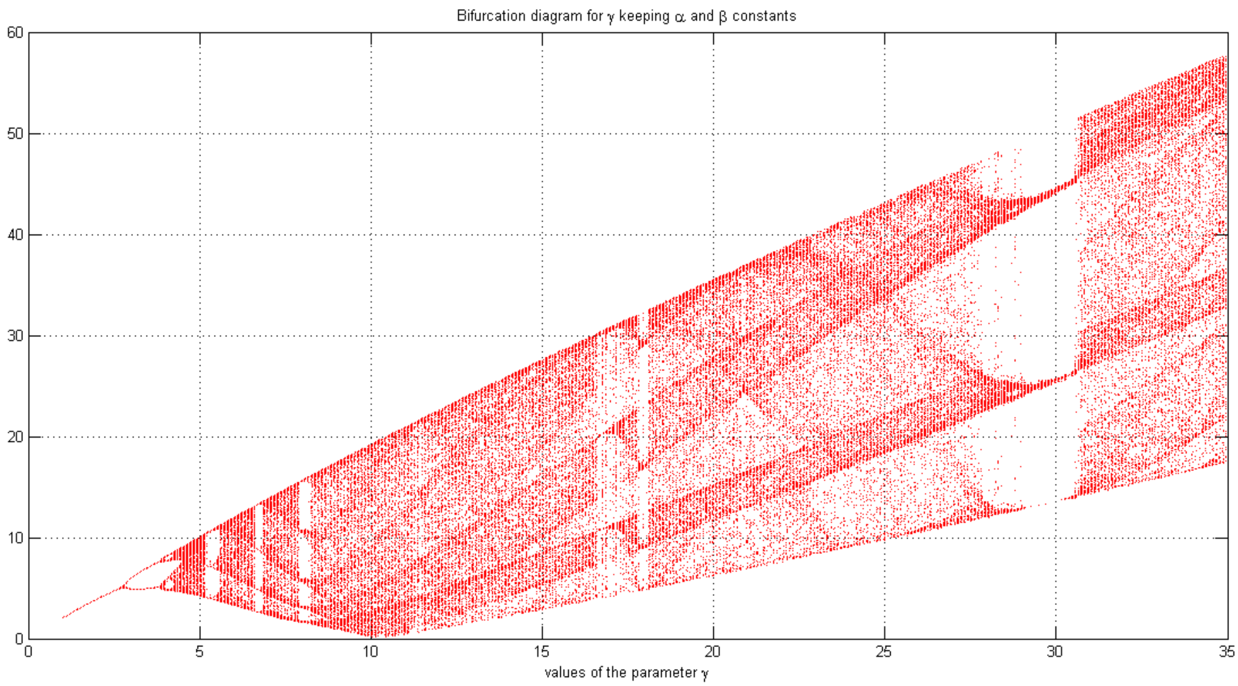

Fig. 5: Bifurcation diagram for γ

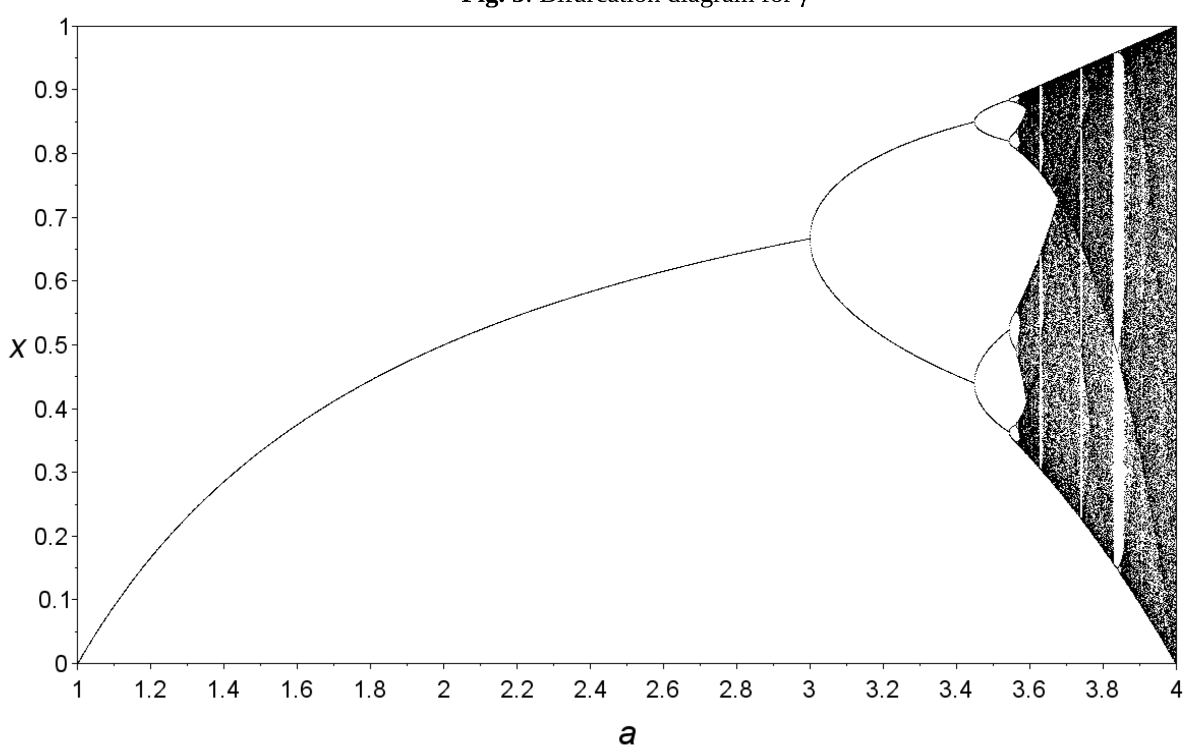

Fig. 6: Bifurcation diagram for the logistic family

3.3 Strange Atrractor

When the parameter γ takes the value 6.3, a strange attractor is observed. The strange attractor is as shown in figure 7. Observing the strange attractor, we notice that the solution curves are spirled in the *xy*-plane and then they escape out in the third dimension without intersecting with the curves in the *xy*-plane. The phase plane of the limit cycles in the three dimensional space for different values of the parameter γ can be ibserved in figure 7.

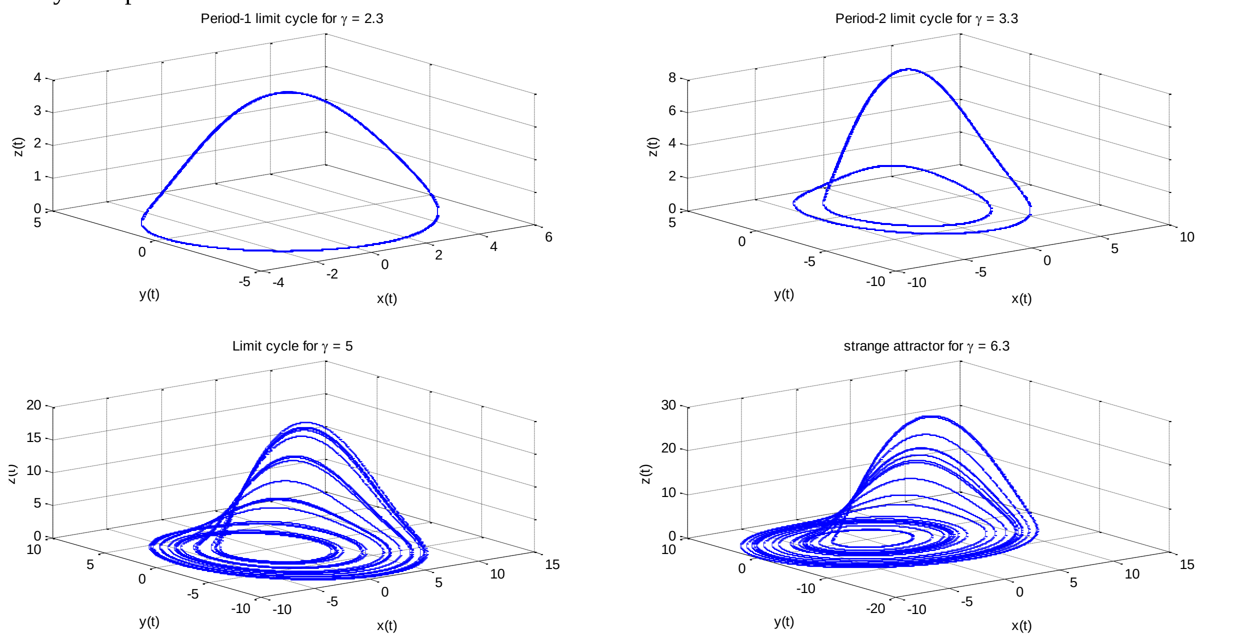

Fig. 7: limit cycles in three dimensional space

The presence of the strange attractor is one of the features of the chaotic systems. The strange attractor shows Holf bifurcation from stationary to periodic motion and a series of period doubling bifurcations leading to chaos. It can be observed that the reinjection of the trajectories is faster than the spiraling out of the trajectories fulfilling the Shilnikov criterion of chaos. The projection of the strange attractor in two dimensional spaces and the non periodic nature of the solution curves is as shown in figure 8.

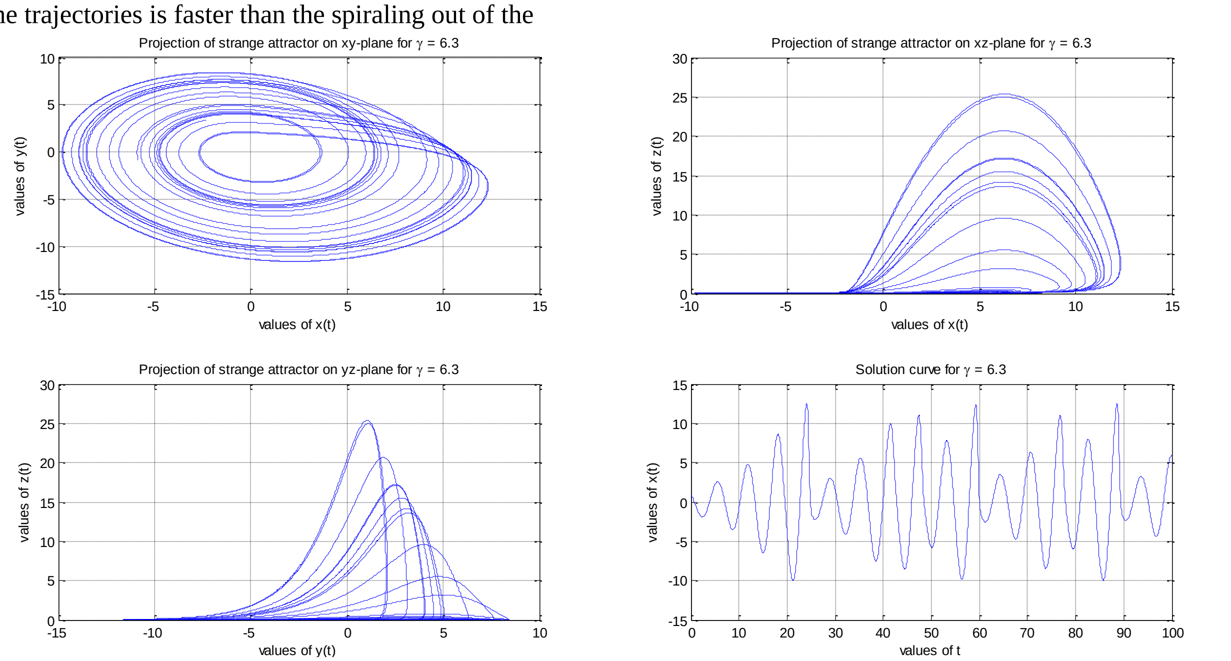

Fig. 8: Projection of strange attractor in 2-dimensional spaces and a solution curve

A strange attractor can also be found in a two dimensional quadratic mapping the Henon map $f: R^2 \rightarrow R^2$ which is defined by $f(x, y) = (1 - \alpha x^2 + y, \beta x)$, where α and β are parameters. By Kulkarni P R [15], it has been proved that keeping $\beta = 0.4$ fixed and varying the other parameter α, a period 1-cycle is observed for $\alpha = 0.2$, a period 2-cycle is observed for $\alpha = 0.5$, a period 4-cycle is observed for $\alpha = 0.9$ and so on. As α becomes greater than 1.01, this period doubling behavior is not observed, showing thereby non-periodic limit cycles. For $\alpha = 1.2$, a strange attractor is observed.

3.4 Poincare Sections

When we slice a strange attractor by a plane, the resulting two dimensional picture is known as the Poincare section. The Poincare section of the strange attractor for the parameter value $\gamma = 6.3$ is as shown in figure 9. In the Poincare section of the strange attractor, we can observe an infinite set consisting of points representing trajectories that escape out in the third dimension with certain gaps in between them. This kind of section has a fractal dimension and it is a Cantor set.

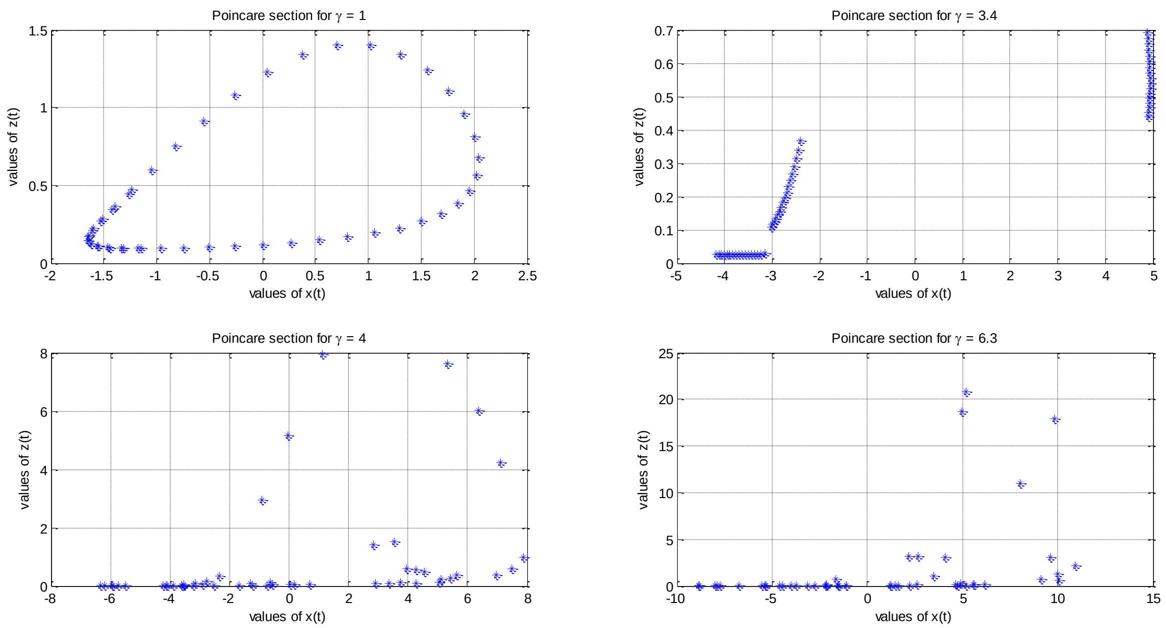

Fig. 9: Poincare sections

3.5 Sensitivity to Close Initial Conditions

Sensitive dependence on initial conditions means that on a given set of initial conditions which vary with neglisible quantities, the trajectories or motions of the system shows a totally different behavior as the time passes. Figure 10 shows that with two sets of initial conditions $x(0) = 1, y(0) = 1, z(0) = 0$ and $x(0) = 0.9, y(0) = 1, z(0) = 0$, the two trajectories though show similar pattern for a short time period, but as the time goes on increasing, they diverge rapidly and after a long period of time, they have different features. In figure 10, we can observe that there are different kind of oscillations and humps as the time t passes the value 110. This sensitive dependence on initial conditions indicate that the long term prediction of the neraby trajectories is practically impossible in such systems where the small differences in the initial conditions are amplified on a large scale as the system evolves over time.

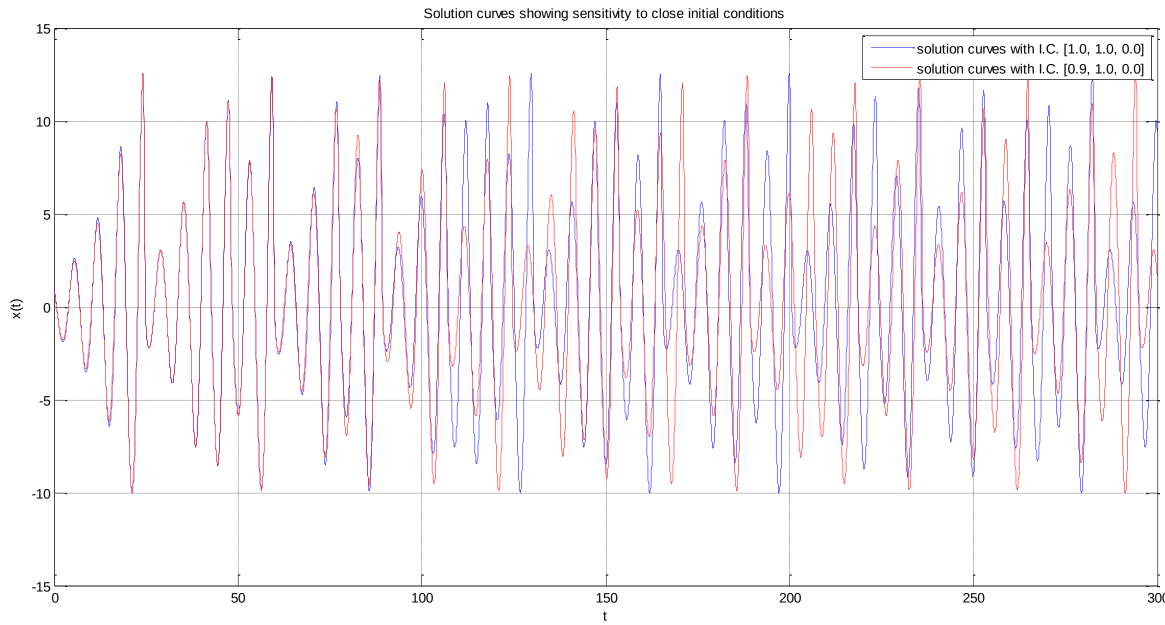

Fig. 10: Sensitive dependence on initial conditions

4. CONCLUSIONS

Most of the phenomenon occurring in natural systems, non-linear motions, mechanical vibrations [13, 16] etc. exhibit a very strange characteristic known as *chaos*. Although there is no worldwide accepted definition of chaos among scientists, there are certain common features that are shown by chaotic systems and which the most of the scientists agree upon. Chaos is the behavior of a system where there is predictability or periodic motion for certain parameter values, but as the parameter values change or as there is a small change in the initial conditions, or as the system evolves over time, this predictable nature of the system turns out to be random or unpredictable one. Some of the common feature of a chaotic system are as follows.

1. Most of the systems which are non-linear in nature show chaotic behavior.
2. Predictability over a short period of time.
3. Sensitivity on initial conditions.
4. Unpredictable long term behavior.
5. Cascade of infinitely many times period doubling.
6. Appearance of the strange attractor.
7. Fractal dimension of the strange attractor.
8. Positive Lyapunov constant.
9. Fractal dimension of Poincare section of the strange attractor.

Most of these features are observed in case of the Rossler system and thus, we have shown that the Rossler system is chaotic in nature. Many ideas can be utilized to verify that a discrete one dimensional one parameter family of mappings is chaotic. It can be observed from the bifurcation diagram that for an approximate value of $\gamma = 5.3$, there is a period three limit cycle. A well-known paper by James Yorke and T-Y Li [7] also guarantees the chaos in the Rossler system. The author Kulkarni P. R.[11, 14] has obtained the values of the parameter c for which the family of mappings $f_c(x) = x^2 - x + c$ has a period-3 orbit and thus proved that it is chaotic. Chyi-Lung Lin and Mon-Ling Shei[1] have proved that the logistic family of mappings $f(x) = \mu x(1-x)$ is topologically conjugate to the mapping $f(x) = (2-\mu)x(1-x)$ and hence is chaotic in nature. Authors Kulkarni P. R. and Borkar V. C.[12] have obtained a topological conjugacy of the family of mappings $f_c(x) = x^2 - x + c$ with the shift mapping σ and proved the chaotic nature of $f_c(x) = x^2 - x + c$ in the sense of Devaney R. L.[8]. Controlling of chaos one of the most challenging tasks before researchers all over the world as it is too difficult to have predictions about the exact parameter values for which a system falls into a chaotic regime.

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SYNTHESIS AND CHARACTERIZATION OF NOVEL ORGANIC NONLINEAR OPTICAL CRYSTAL: L-PHENYLALANINIUM TARTRATE

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Abstract. *L-Phenylalaninium Tartrate (LPT) organic NLO crystal was grown first of its time and subjected to various characterizations. Single crystal XRD studies were conducted. UV-visible spectral studies were carried out. Absorbance and transmittance spectra of LPT crystal were recorded. From the spectrum it is clear that the LPT crystal has a UV cut-off of approximately 265 nm. The band gap energy of the grown LPT crystal was calculated using the Tauc's plot and it is found to be 4.65 eV. PL spectrum of LPT crystal was recorded by exciting with UV light at 250 nm. The PL emission spectrum of LPT crystal contains the emission peaks at 468 and 543 nm in the visible region of the spectrum and the peak at 828 nm is due to the emission of IR radiation. FTIR spectral method was used to identify the presence of functional groups in the grown crystal. Microhardness study was performed by using a Microhardness analyser. Impedance analysis was carried out for LPT crystal by using an impedance analyser at different frequencies. From dielectric studies, it is observed that, the dielectric parameters like dielectric constant and loss factors decrease with increase in frequency and their values increase with increase in temperature. From the SHG measurement, it is observed that there is a green laser light emitted from the sample and the relative SHG efficiency of LPT sample is 1.71 times that of KDP sample. Z-scan technique was used to determine third-order NLO parameters of LPT crystal. LDT value was determined. The obtained value of LDT of LPT crystal is 2.675 GW/cm^2.*

Keywords. *single crystal; organic material; solution method; XRD; optical transmittance; micro-hardness; impedance; NLO; LDT;*

1. INTRODUCTION

Nonlinear Optical (NLO) materials play a major role in both academic research and technological applications. NLO crystals are the most suitable materials for frequency conversions, second harmonic generation, laser technology, optical data storage, optical switching and fibre optic communications [1]. Organic crystals possess higher non-linear efficiency compared to inorganic crystals. The organic crystals are effectively applied in active research due to its higher non-linear second order coefficients [2]. Amino acids play a promising role in NLO Applications as they possess proton acceptor amino group (NH_3^+) and a proton donor carboxyl group (COO^-) [3]. L-Phenylalanine is an amino acid, with surprisingly good SHG efficiency in it [4]. L-tartaric acid is a white crystalline organic material that occurs fruits, grapes, bananas, citrus and tamarinds and it forms many tartrate compounds [5].

2. EXPERIMENTAL METHOD

The organic single crystal of L-Phenylalaninium Tartrate was grown by dissolving high purity L-Phenylalanine and Maleic acid in 1: 1 molar ratio in a beaker containing 100 ml of double distilled water. The solution was stirred about 5 hours and heated in a constant temperature of about 50° C. The prepared solution was filtered by Whatmann filter paper and left for slow evaporation at room temperature. Initially, seed crystals were obtained. By placing some of the good quality seed crystals in the saturated solution, big-sized crystals of LPT were grown. The schematic diagram of crystal growth process for growing LPT crystal is shown in figure 1 and the grown crystal is shown in figure 2. A good quality of LPT crystal of 25 × 8 × 7 mm^3 size was obtained within one month. It is seen that the grown crystal is transparent and colourless.

3. RESULTS AND DISCUSSION

3.1 Single crystal XRD studies

The grown crystal of LPT is confirmed by single crystal XRD studies using ENRAF NONIUS CAD-4 diffractometer with MoK$_\alpha$ radiation (λ=0.71073 Å) to determine the crystal structure and lattice constants. From the analysis, the structure of the grown crystal is observed to be orthorhombic and the data are compared with the parent compound and are presented in Table 1.

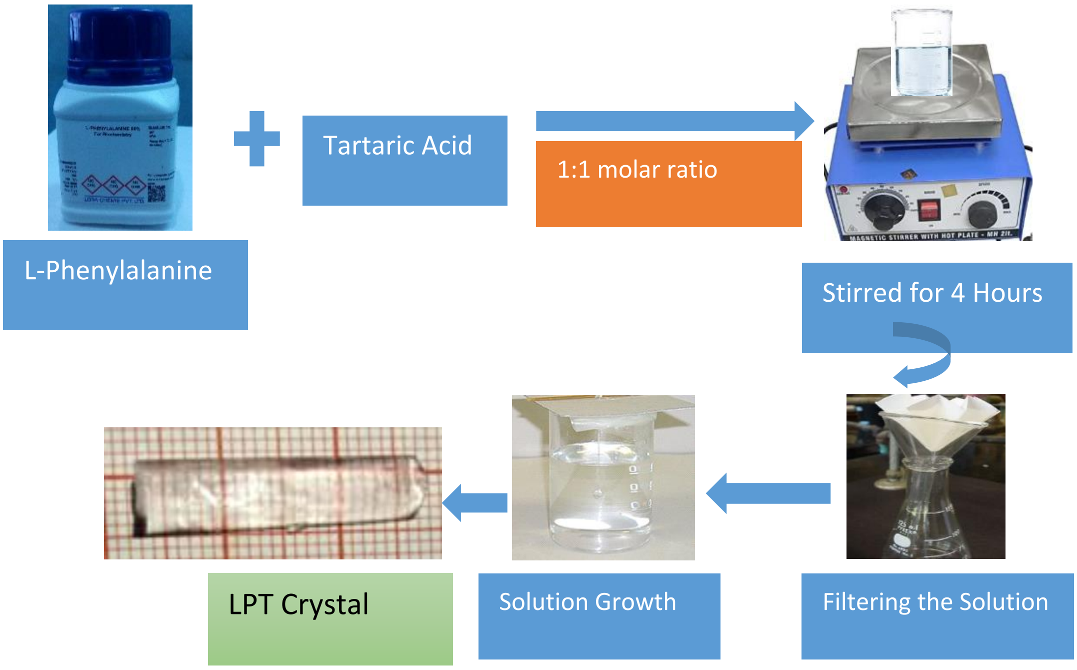

Fig: 1 Schematic diagram of Crystal growth process for growing LPT crystals

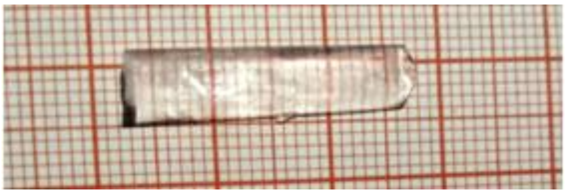

Fig.2 The grown bulk single crystal of LPT

Table 1 Unit cell parameters of LPT crystal

Parameters	**LPT crystal**
Crystal System	Orthorhombic
α	90º
β	90º
γ	90º
a (Å)	6.431 (2)
b (Å)	7.583 (4)
c (Å)	12.694(1)
Unit cell volume(Å^3)	619.04 (3)

3.2 UV-visible spectral studies

The optical properties of the grown crystal are studied by using UV-Vis analysis. The instrument used for recording UV spectrum is Perkin Elmer Lambda 35 spectrophotometer.

A graph of Absorbance vs Wavelength is drawn. It is shown in the figure 3. From the spectrum, it is clear that, the LPT crystal has a UV cut-off of approximately 265 nm. It reveals that, the grown crystal has good transmittance of nearly 80%, without the absorption peak in the entire visible region [6].

The Transmittance versus Wavelength graph was drawn and indicated by figure 4. From the graph, it is clear that the crystal has a wide transparency range, which enables it as a good candidate for NLO applications especially optoelectronic applications [7,8].

The band gap energy of the grown LPT crystal was calculated using the Tauc's plot and it is given in figure 5. The band gap is found to be 4.65 eV. This indicates that the LPT crystal is a higher band gap energy material.

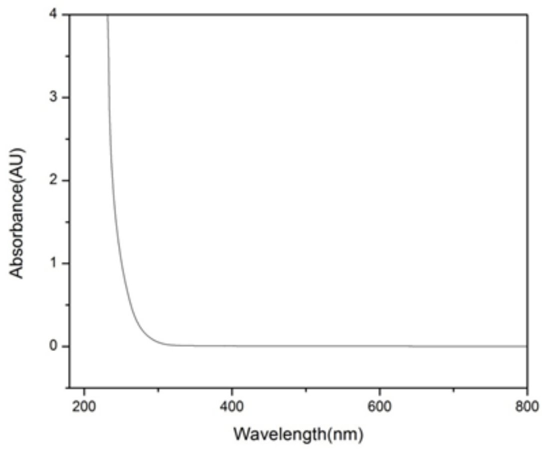

Fig. 3 UV-Vis absorbance spectrum of the grown LPT Crystal

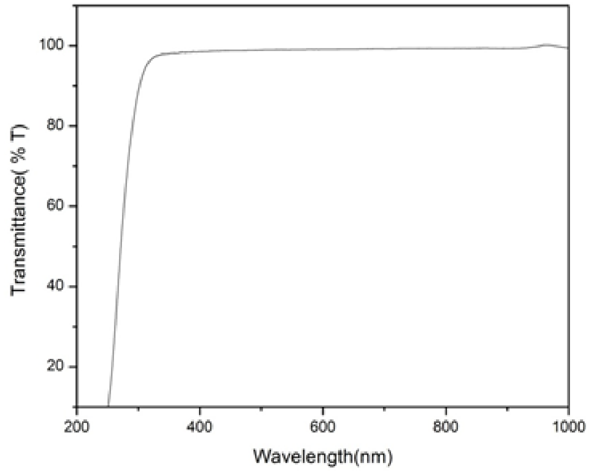

Fig. 4 UV-Vis transmittance graph of the grown LPT crystal

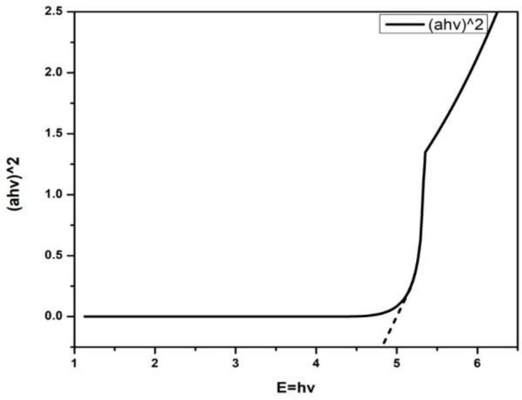

Fig. 5 Tauc's plot of the grown LPT crystal

3.3 Photoluminescence study

Photoluminescence (PL) is a phenomenon which generates optical radiation in UV, visible and IR spectral ranges when a sample is excited with UV light. PL is the emission of light radiation during the transition from its lowest vibrational energy level of the excited state back to its ground state. The loss of emission of photons is due to the vibrational relaxation, internal conversion and intersystem crossing. PL spectrum of LPT crystal was recorded at room temperature by exciting the crystal with UV light at 250 nm. The PL emission spectrum of LPT crystal is shown in the figure 6. From the result, it is observed that there are emission peaks at 468 and 543 nm in the visible region of the spectrum. The emission peak at 828 nm is due to the emission of IR radiation. The optical and electronic properties of the grown LPT crystal are studied by using photoluminescence analysis [9].

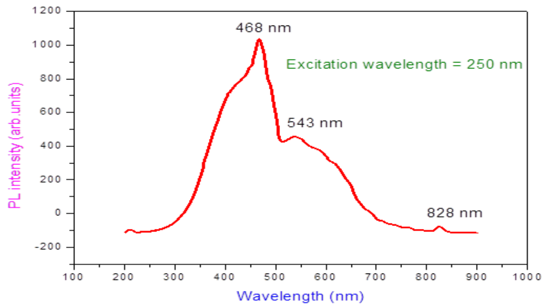

Fig. 6 Photoluminescence spectrum of the grown LPT crystal

3.4. FTIR spectroscopy

FTIR analysis was carried out to identify the presence of functional groups in the grown crystal. The characteristic absorption peaks was recorded in 400 – 4000 cm^{-1} using the Perkin Elmer FTIR Spectrometer and it is given in figure 7. The peak at 3168 cm^{-1} indicates the NH_3^+ stretching vibration and the peak at 3028 cm^{-1} corresponds to NH vibrational stretch [10]. The peak at 2048 cm^{-1} indicates the CH stretching. The peak at 1416 cm^{-1} indicates the presence of C=O stretch of COOH group [11]. The peak at 1347 cm^{-1} denotes the OH bend of COOH group [12]. The peak at 1041 cm^{-1} corresponds to C-H rocking vibration. The peak at 701 cm^{-1} indicates the in plane deformation of COO^- group.

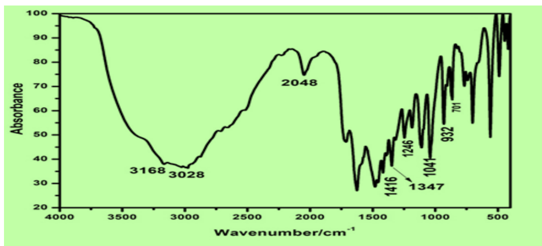

Fig. 7. FTIR spectrum of the grown LPT crystal

3.5 Microhardness studies

Measurement of hardness is a useful non-destructive testing method used to determine the applicability of the crystal in the device fabrication and it is one of the best methods to understand the mechanical properties of materials [13]. Crystals, free from cracks, with flat and smooth faces are chosen for Vickers Microhardness test [14]. For different loads of 25 g, 50 g, 75 g and 100 g, the indentations were applied at a constant indentation time with an interval of 25 s. Diagonal lengths of indentation (d) were noted in μm for different applied load (P) in g. The variation of diagonal length with applied load for LPT crystal is shown in figure 8. The Vickers hardness (H_V) number at different loads were calculated [15] using the following relation:

$$H_V = \frac{1.8544\,P}{d^2} \qquad (1)$$

Figure 9 shows the variation of Vickers hardness number (H_V) with the applied load (P). It is seen that the hardness number is increasing while increasing the applied load. Beyond 75 g, there is a slight decrease of hardness. It is because of formation of a crack on the surface of the sample.

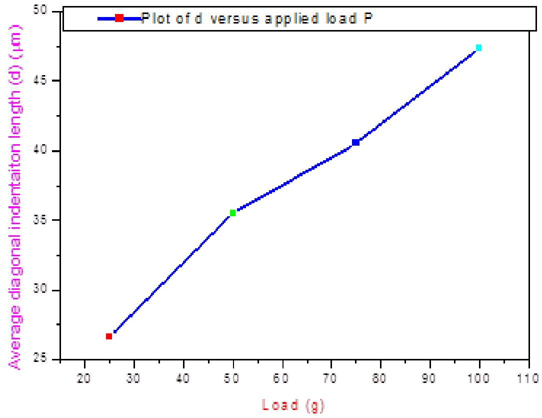

Fig.8 Variation of diagonal length with applied load for LPT crystal

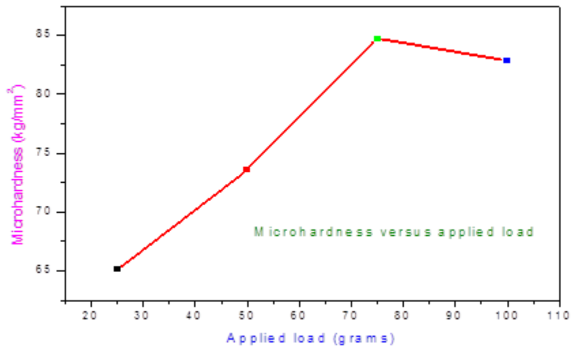

Fig.9 Variation of Microhardness with applied load for LPT crystal

Based on Meyer's analysis of hardness, relation connecting load P and indentation length d is

$$P = k_1 d^n$$

Where k_1 is the material constant, n is the Meyer's index or work hardening coefficient. Taking log on both sides of the above equation, it becomes

$$log P = log k_1 + n\, log d$$

A Plot of log P versus log d is shown in figure 10 and it yields a straight line graph. Its slope gives the value of work hardening coefficient (n) and it is found to be 3.102.

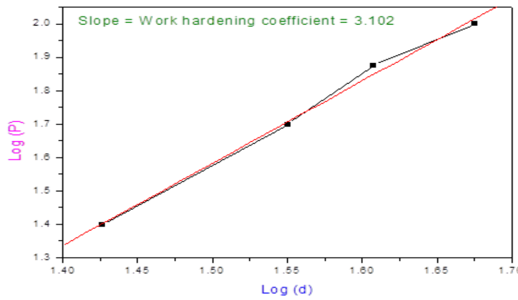

Fig. 10 Plot of log (d) versus log (P) for LPT crystal

According to Onitsch [16] and Hanneman [17], n value lies between 1 and 1.6 for hard materials and it is more than 1.6 for soft materials. Hence, LPT crystal belongs to soft material category. Kick's law states that since the material takes some time to revert to the elastic mode after every indentation, a correction term has to be applied to the d-value [18], and it is given by

$$P = k_2 (d + x)^2$$

$$d^{n/2} = \left(\frac{k_2}{k_1}\right)^{1/2} d + \left(\frac{k_2}{k_1}\right) x$$

A graph is drawn between $d^{n/2}$ and d is shown in figure 11. This graph yields a straight line tendency with the slope of $(k_2/k_1)^{1/2}$ and with an intercept of $(k_2/k_1)\ x$. The obtained values of k_1, k_2 and x are tabulated in Table 2.

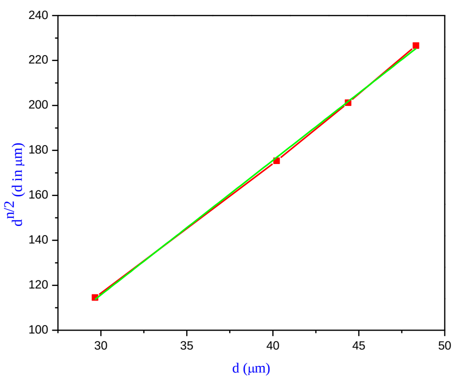

Fig. 11 Plot of $d^{n/2}$ versus d

Table 2 Values of the constants such as n, k_1, k_2 and x

Parameter	Value
n	3.1020
K_1 (kg/mm)	0.00251
K_2 (kg/mm)	0.05630
x	-7.4920

3.6. Impedance analysis

Impedance is defined as the frequency domain ratio of the voltage to the current and it is the opposition of the flow of alternating current (AC) in a complex system [19]. Impedance analyses were carried out for the grown crystal to find out the impedance, bulk resistance, capacitance, DC conductivity and relaxation time [20]. This technique analyzes the ac response of a system to a sinusoidal perturbation and subsequent calculation of the impedance as a function of frequency of the perturbation [21]. The frequency dependent electrical properties of a material are often represented in terms of complex impedance Z* = Z' + jZ" where Z' is the real part of impedance and Z" is the imaginary part of impedance [22]. Figure 12 gives the variation of real part of impedance with frequency at room temperature for LPT crystal. From the results, it is observed that the real part of impedance decreases with rise in frequency [23]. The high real part of impedance at low frequency indicates low ion mobility in the grown sample and it may result in improving NLO properties of the sample [24]. Figure 13 presents the variation of imaginary part of impedance (Z") with frequency at room temperature for the grown crystal [25]. The Nyquist plot for LPT crystal is drawn between Z' versus Z" and it is shown in the figure 14.

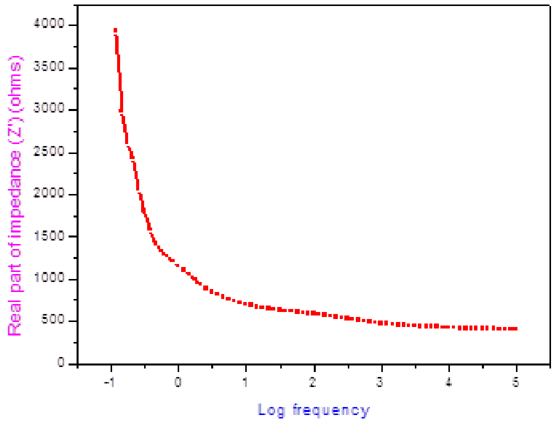

Fig.12 Variation of real part of impedance with frequency at room temperature for LPT crystal

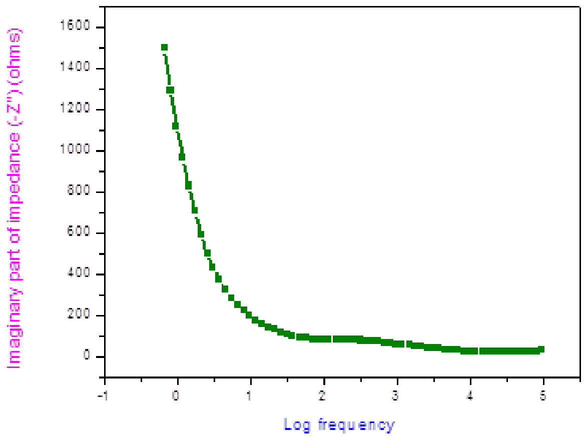

Fig.13 Variation of imaginary part of impedance with frequency at room temperature for LPT crystal

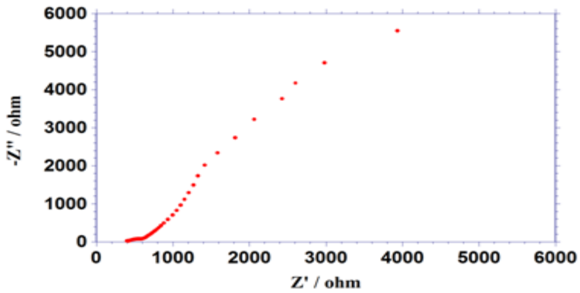

Fig.14 Nyquist plot for LPT crystal

3.7. Dielectric measurement

Dielectric studies were carried out to investigate the response of the crystal to an applied electric field and to determine various electrical parameters such as dielectric constant (ε_r), dielectric loss (tan δ) and conductivity at different temperatures [26]. Occurrence of a dielectric between the plates of a condenser increases the capacitance. Essentially, dielectric constant is the measure of how easily a material is polarized in an external electric field [27]. Dielectric parameters depends on the frequency applied and temperature. The variations of dielectric parameters of the samples with temperatures and frequencies are presented in the figures 15 and 16. From the graphs, it is observed that dielectric parameters like dielectric constant and loss factor decrease with increase in frequency and their values increase with increase in temperature. The high values of ε_r at low frequencies may be due to presence of space charge polarization. Its low value at high frequencies is because of the loss of four polarizations viz. space charge, orientational, ionic and electronic polarization. It is to be noted here that, the space charge polarization is dominant whereas electronic and ionic polarizations are not very much active in low frequency range and the low value of dielectric constant at higher frequencies will be due to the reason of the slagging of dipoles with respect to the quick changes in the applied field. It is a suitable parameter for the enhancement of SHG coefficient and extending the samples application towards photonic, electro-optic and NLO devices [28]. Increase of dielectric constant with temperatures may be due to the thermal excitation of atoms about their lattice point and blocking of charge carriers at the electrodes [29]. It is observed that at lower frequencies and higher temperatures, the dielectric constant and dielectric loss of the sample is larger [30]. High dielectric constant values of the sample leads to power dissipation. A material having low dielectric constant will have less number of dipoles per unit volume and as a result, it may have minimum loss compared to the material having high dielectric constant.

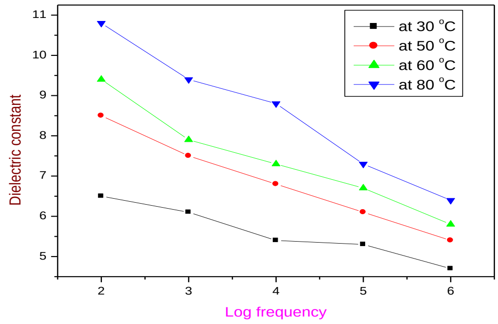

Fig.15 Variation of dielectric constant with frequency at different temperatures for LPT crystal

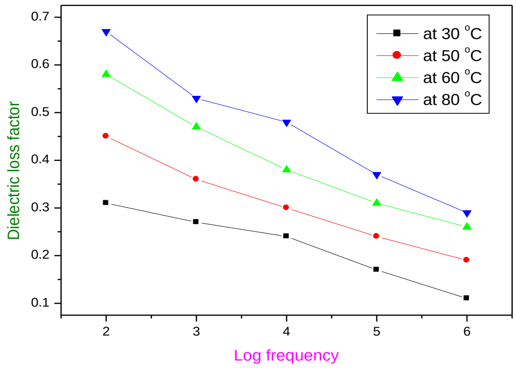

Fig.16 Variation of dielectric loss with frequency at different temperatures for LPT crystal

3.8. Second-order NLO studies

Second harmonic generation (SHG) is an optical process that results in the conversion of an input optical wave into an output optical wave with a frequency twice that of the input wave. This frequency doubling process is particularly used to make green laser light with a wavelength of 532 nm from an Nd: YAG laser operates at 1064 nm. Kurtz and Perry powder method is used for analyzing the SHG efficiency of the sample [31]. The laser is focused on a powdered sample, and the light emitted is collected, filtered and detected using a photomultiplier tube [32]. The primary laser beam with a wavelength of 1064 nm, and a pulse width of 6 ns, and a pulse rate of 10 Hz are made to fall on the powdered LPT crystal sample. In this experiment, the reference sample used is KDP [33]. The obtained data from the SHG experiment for the LPT sample are given in table 3. From the SHG data, it is observed that there is a green laser light emitted from the sample, and hence LPT sample shows the second-order NLO effect. The result indicates that the relative SHG efficiency of LPT sample is 1.71 times that of KDP sample [34].

Table 3 SHG data for LPT crystalline material

Sl. No.	Sample Code / Name of the sample	Output Energy (milli joule)	Input Energy (joule)
1	KDP (Reference)	8.90	0.70
2	LPT sample	15.22	0.70

3.9. Third-order NLO studies-Z-scan technique

There are two modes of measurement in the Z-scan technique, viz. open aperture and closed aperture modes. A He–Ne laser (λ = 632.8 nm) is utilized as the light source in this measurement. The light intensities are determined as a function of sample location in the Z-direction relative to the focal plane using closed or open aperture techniques to resolve the nonlinear refraction and absorption coefficients [35]. The relation between transmission difference between peak and valley (ΔT_{p-v}) from closed aperture Z-scan curve and the phase shift ($\Delta\phi$) is given by

$$\Delta T_{p-v} = 0.406(1-S)^{0.25}|\Delta\phi|$$

Here S is the linear transmittance aperture. Using the above relation, the phase shift ($\Delta\phi$) is determined, and using this value, the third-order nonlinear refractive index (n_2) is calculated using the following relation

$$n_2 = \Delta\phi \;/\; K I_0 L_{eff}$$

where I_o is the intensity of the laser beam at the focus, L_{eff} is the effective thickness of the sample, and K is the wave vector [36]. In the closed aperture Z-scan curve, the nonlinear absorption coefficient (β) can be determined using the following relation

$$\beta = 2\sqrt{2}\Delta T \;/\; I_0 L_{eff}$$

Where ΔT is the peak value of the Z-scan curve for an open aperture.“ The value of β will be negative in the case of saturable absorption and positive in the case of two-photon absorption”[37]. The following relations can be used to derive the real and imaginary components of the third-order nonlinear susceptibility **($\chi^{(3)}$).**

Real part of $\chi^{(3)} = (10^{-4}\varepsilon_0 c^2 n_0^2 n_2^2) \;/\; \pi$

Imaginary part of $\chi^{(3)} = (10^{-2}\varepsilon_0 c^2 n_0^2 \lambda\beta) \;/\; 4\pi^2$

Where ε_o is the permittivity of free space or vacuum, n_o is the linear refractive index of the sample, λ is the wavelength of light and c is the velocity of the light [38-39].

The open aperture and closed aperture Z-scan curves for LPT crystal are shown in figures 17 and 18 respectively. Since the closed aperture curve shows a peak followed by a valley, LPT crystal has the negative value of the nonlinear refractive index, and it is due to non self-defocusing nature. The important values in connection with the Z-scan analysis are provided in table 4. From the obtained data, the nonlinear third-order optical susceptibility of LPT crystal is observed to be high and hence this crystal could be useful in NLO applications.

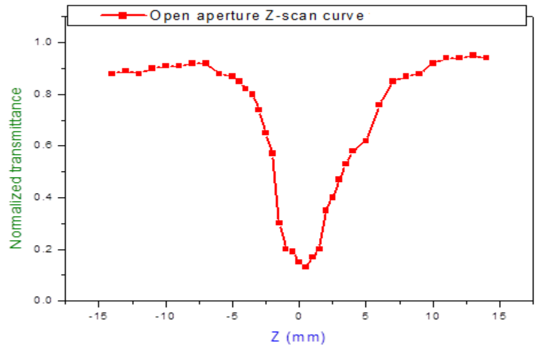

Fig. 17 Open aperture Z-scan curve of LPT crystal

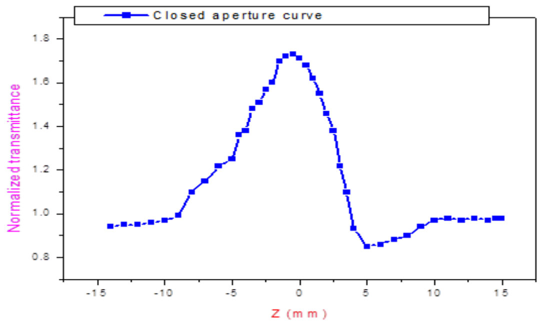

Fig. 18 Closed aperture Z-scan curve of LPT crystal

Table 4 Z-scan data for LPT crystal

Important parameters	**Values**
Laser wavelength (λ)	632.8 nm
Lens focal length	30 mm
Aperture radius (r_a)	4.1 mm
Spot size in front aperture (ω_a)	4.5 mm
Incident intensity	2 MW/cm^2
Sample thickness	0.77 mm
Nonlinear absorption coefficient (β)	3.024×10^{-5} m/W
Nonlinear refractive index (n_2)	- 6.247×10^{-9} m^2/W
Real part of third-order susceptibility	5.402×10^{-7}esu
Imaginary part of third-order susceptibility	7.551×10^{-8}esu
Third-order susceptibility, $\chi^{(3)}$	5.448×10^{-7}esu

3.10 Laser damage threshold measurement

An important related property of NLO crystals is the threshold for catastrophic laser induced damage. Laser induced damage in optical materials is a phenomenon involving interaction of high power laser radiation with matter and various physical, chemical, mechanical, optical and other aspects of materials that come into play. It is evident that the harmonic conversion efficiency is proportional to the power density of the fundamental beam. Hence, a convenient way to increase the efficiency is to focus the beam into the crystal. But, this often leads to breakdown of the materials, catastrophically damaging the crystal. It is then useful to prescribe the maximum permissible power for a particular crystal, defined as damage threshold.

The minimum power level which causes damage to at least 50% of irradiated sites is defined as the single shot damage threshold. Laser damage threshold is a special parameter of a material. Before applying a crystal as an NLO component in various applications such as frequency doubling, optical parametric processes etc., one should calculate the LDT value of the particular crystal [40]. LDT measurement was conducted for the crystal at 1064 nm. The laser damage threshold depends on pulse duration, focal spot geometry, sample quality, previous history of the sample, experimental technique employed etc. The experimental set-up used for the measurement of laser damage of the samples is shown in figure 19. A Q-switched Nd: YAG laser (Continuum USA, Model: Surelite-III) of wavelength 1064 nm and pulse width of 10 ns was used. The energy of the laser pulse was controlled by an attenuator (combination of λ/2 plate and polarizer) and delivered to the test sample located near the focus of a plano-convex lens of focal length 30 cm. The presence of single pulse damage was found by checking the fall of transmitted intensity as determined by a fast PIN type Si photodiode and drawn in a digital storage oscilloscope (Tektronix:TDS 3054B). A pyro-electric energy meter was used for measuring the energy of the input laser pulse for which the crystal gets damaged. The LDT value was determined using the formula $P = E/\pi\tau r^2$ where E is the input energy in mJ, τ is the pulse width in ns and r is radius of the laser spot in mm [41, 42]. The calculated value of LDT of LPT crystal is 2.675 GW/cm^2. Since this value of high, LPT crystal could be used for laser applications [43].

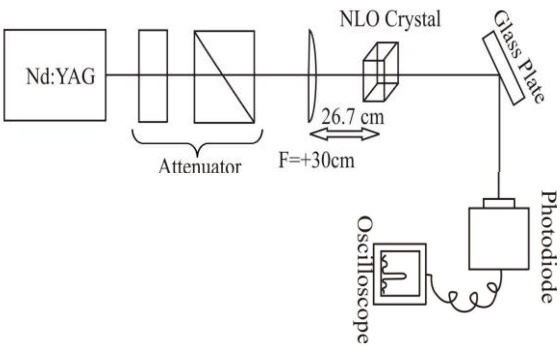

Fig. 19 Block diagram for the laser damage threshold measurement

4. CONCLUSIONS

In the present study, an organic NLO crystal viz. L-Phenylalaninium Tartrate (LPT) was grown first of its time and subjected to various characterizations. The grown crystal of LPT is subjected to single crystal XRD studies. From the analysis, the structure of the grown crystal is found to be orthorhombic. UV-visible spectral studies were carried out by using a spectrophotometer. Absorbance and transmittance spectra of LPT crystal were recorded. From the spectrum it is clear that the LPT crystal has a UV cut-off of approximately 265 nm, reveals that the grown crystal has good transmittance of nearly 80% without the absorption peak in the entire visible region. The band gap energy of the grown LPT crystal was calculated using the Tauc's plot and it is found to be 4.65 eV. PL spectrum of LPT crystal was recorded by exciting with UV light at 250 nm. The PL emission spectrum of LPT crystal contains the emission peaks at 468 and 543 nm in the visible region of the spectrum and the peak at 828 nm is due to the emission of IR radiation. FTIR spectral method was used to identify the presence of functional groups in the grown crystal. The functional groups identified are NH_3^+, NH, CH, C=O, CN, COO^- etc. Microhardness study was performed by using a Microhardness analyser and it is found that hardness increases with increase in applied load up to 75 g and then it decreases. Meyer's law was used to calculate the value of work hardening coefficient and it is found to be 3.102. Impedance analysis was carried out for LPT crystal by using an impedance analyser at different frequencies. From the results, it is observed that the real part of impedance decreases with rise in frequency and also imaginary part of impedance decreases with increase in frequency. The Nyquist plot for LPT crystal was also drawn between Z' versus Z". From dielectric studies, it is observed that, the dielectric parameters like dielectric constant and loss factors decrease with increase in frequency and their values increase with increase in temperature. The high values of ε_r at low frequencies may be due to the presence of space charge polarization and its low value at high frequencies may be due to the loss of four different type of polarizations such as space charge, orientational, ionic and electronic polarization. The low value of dielectric constant at higher frequencies will be due to the reason of the slagging of dipoles with respect to the quick changes in the applied field. From the SHG measurement, it is observed that there is a green laser light emitted from the sample and the relative SHG efficiency of LPT sample is 1.71 times that of KDP sample. Z-scan technique was used to determine third-order NLO parameters of LPT crystal by using a He–Ne laser (λ = 632.8 nm). From the obtained data, the nonlinear third-order optical susceptibility of LPT crystal is observed to be high and hence this crystal could be useful in NLO applications. The LDT value was determined. The obtained value of LDT of LPT crystal is 2.675 GW/cm^2. Hence the grown LPT crystal shall be used in Laser applications also.

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A STUDY OF SYNTHESIS AND CHARACTERISATION OF BI_2TE_3-PANI WITH AND WITHOUT SELENIUM DOPING

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Abstract. *In this paper, our main objective is to have a methodical study of production and thermoelectric property of Bi_2Te_3-PANI with and without Selenium doping. A comparison has drawn between the thermoelectric properties of Bi_2Te_3-PANI with that of the pure Polyaniline (PANI). Pure PANI as well as Bi_2Te_3-PANI composite was synthesized with the help of chemical oxidative technique. The thermal and electrical characteristics have been studied for both doped and undoped Bi_2Te_3-PANI and it has been seen the thermal conductivity has been decreased for the doped sample. On the other hand, the electrical conductivity and conduction power is high in the case of doped Bi_2Te_3-PANI in comparison to pure PANI. All these results leads us to the conclusion that Selenium doped Bi_2Te_3-PANI has the characteristics of perfect thermoelectric material.*

Key works: *Bi_2Te_3-PANI, Conductivity, Doping, Figure of merit, Thermoelectric property*.

1. INTRODUCTION

One of the most feasible substitute energy sources for today's world is thermoelectric energy. This no pollution energy source is able to trim down our reliances on carbon based fossil fuels and nuclear energy [1, 2]. All the alternative heat energy sources like solar, geothermal etc. can be transformed into electricity by using thermoelectric energy devices which are easy to construct and handle. Thermoelectric materials have been used in thermoelectric (TE) devices which obtain electricity from ravage heat emitted from industry and vehicles[3, 4]. Only 30-40% heat energy has been used to generate of electricity and rest of energy input is lost as ravage heat. Thermoelectric materials can transform ravage heat into electricity. This conversion efficiency from heat into electricity is related to intrinsic electrical and thermal properties. Electrical conductivity is a measure of how well electrical current can pass through a material under the influence of an applied voltage/electric field. Thermal conductivity measures how well heat can pass through a material under a temperature differential.

The efficiency of thermoelectric materials is related to a dimensionless quantity, the thermoelectric figure of merit ZT, which can be defined as

$$ZT= (\sigma S^2)\,T/\kappa = PT/\kappa \qquad (1)$$

Where σ represents the electrical conductivity, S represents the Seebeck coefficient, κ represents the thermal conductivity, and T represents the absolute temperature. The product σS^2 has been called as the TE power factor P[4, 5]. Therefore, by increasing S while keeping the optimum, the values that increase the power factor (σS2) are prime which can be termed as the important approach for obtaining high efficiency thermoelectric material.

At present, all of the hard works are done on modifying the TE properties by different methods, like doping [6], nanostructuring, nanocomposite formation, and molecular rattling, and hence Polymer composites becomes an important part in this industry[7]. The properties shown by conducting polymers at very little temperature has many uses namely minimum price of manufacture, low weight, and flexibility[8].

Several scientists have studied Polyaniline (PANI) [9]. PANI is particularly important as it is fairly low-cost and has three separate oxidation states with special colors and also it responds to acid/base doping. TE can transform heat energy straight into electricity which can be used if necessary. It proposes a capable knowledge to transform heat energy from solar energy. It has also been used to recover ravage heat from manufacturing areas and vehicle exhausts. The TE efficiency of conducting polymer depends on the material[10, 11].

The power factor of TE should be made the most of and the thermal conductivity should be reduced to attain TE materials which have high energy adaptation efficiency[12]. Telluride induced materials usually show high thermoelectric power. As per earlier studies telluride based materials exhibits a TE figure of merit "ZT" upto 2.2, the maximum value claimed. For real life use, such as thermo-electric generators (TEGs) has the value of ZT is up to 3[13, 14].

A number of authors have worked on synthesize and character analysis of Bi_2Te_3 with polyaniline[4]. Mechanical unification technique was used newly by [15] for the production of Bi_2Te_3–PANI. It was established that the power factor of Bi_2Te_3–PANI is lesser than the two individual components. Again, a research work has been published on the production of Bi_2Te_3-PANI mixture by physical addition and mixture displayed a greater power factor[15]. A number of publications are available on TE characteristics of Bi_2Te_3-PANI, experimental research on TE property related to complexes are un-obtainable. With all these literature review, in the current work, we have done a methodical understanding of synthesis and properties of Bi_2Te_3–PANI. We also have studied the thermo-electric characteristic of Bi_2Te_3–PANI after doping using Selenium.

EXPERIMENTAL METHODOLOGY-

Pure polyaniline polymer (PANI) synthesis - Pure polyaniline polymer water was synthesized by chemical oxidation method [16]. Aniline is oxidized with ammonium persulfate in acidic water. Dissolve separately in HCl and double distilled water. Both the liquids were mixed together at room temperature. Then the mixture was stirred for 1 h and allowed to polymerize. The PANI precipitate was collected using filter paper and washed with distilled water HCl and methanol, respectively. PANI powder was dried in vacuum [17].

The Bi_2Te_3 are purchased in the powder form from Sigma company with 99.99% purity.

Synthesis of Bi_2Te_3-PANI - A dilute aniline solution is obtained to prepare these nanocomposites. Bismuth telluride was mixed with the liquid and stirred for 30 min[18]. The color associated with the solution has been changed to black. Now add APS as oxidizing agent and stir the entire mixture at 0-5 °C for 6 h for polymerization. The final mixture is a black-green colored PANI Bi_2Te_3 nanocomposite precipitate formed after vacuum drying in a 5 °C oven for an entire day [19].

DOPING USING Se

Se powder was used to dope Bi_2Te_3-PANI and mix the ingredients in a container. Evenly, using deionized water as solvent, NaOH as pH regulator, $NaBH_4$ It is a reducing agent, EDTA is a coating agent, and it is transferred to In the WDF type high pressure reactor, keep it at 200℃ for 8hr, then cool to room temperature, mixed solution was suction filtered and used repeatedly wash with deionized water and absolute ethanol, and finally vacuum dry at 60℃ After 6 h, Se doped Bi_2Te_3-PANI residue was obtained.

CHARACTERIZATION-

X-ray diffraction (XRD) as well as Fourier transform infrared (FTIR) characterization was used for Bi_2Te_3-PANI and pure PANI. The structures of the sample were analyzed by XRD and FTIR in MNIT, Jaipur. The electrical properties like electrical conductivity were carried out through four probe method in MNIT,Jaipur.

RESULTS AND DISCUSSIONS-

To have the enhanced electric conductivity, we can dope the sample using a adequate quantity of appropriate dopants. The Seebeck coefficient would be increased as we increase the use of dopant to the sample. The reason behind this is the Fermi energy gets enforced profoundly in conduction range with the increase number of charged carriers. Now Selenium (Se) has been considered as the doping agent. First we will consider Bi_2Te_3-PANI composite without any Se doping and after analyzing the different characteristics, we will repeat the work for doped Bi_2Te_3-PANI.

WITHOUT Se DOPING-

Figure no. 1 illustrated XRD patterns for pure PANI, and Bi_2Te_3-PANI. All peak units associated with pure

PANI models are clearly marked. Compared to pure builder, the XRD contour of Bi_2Te_3-PANI combination clearly shows a high peak at the same location. The observation of individual peaks is related to the periodic separation of repeating polyaniline units and the molecular organization of Bi_2Te_3 in the PANI matrix. This indicates that Bi_2Te_3-PANI results in a well-arranged molecular organization of Bi_2Te_3 in triplet combinations.

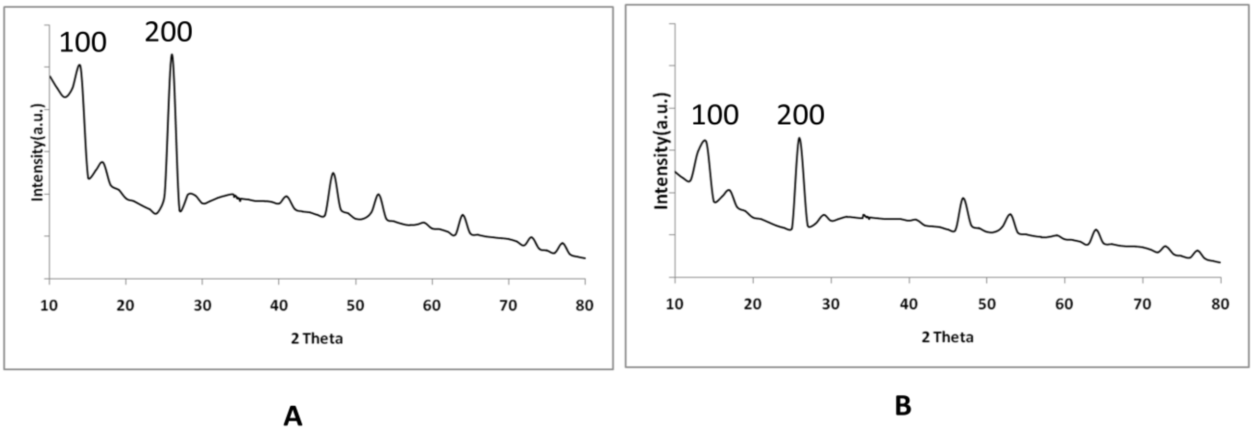

Fig. no. 1- (A) XRD pattern of Bi_2Te_3-PANI (B) XRD pattern of pure PANI.

The Fourier transform of the above mentioned nanocomposite indicated the existence of a doped builder in the nanocomposite infrared spectra. One can see from Fig. no. 2, the peak transmittance at 798 cm^{-1} coincides with the semi-distributed benzene ring which is considered in the literature to be "electronic exploitation". The peaks at 1570.95 cm^{-1} correspond to the extended N-H bond of the benzoid ring (the upper characteristic of the cap) representing that the polymerization reaction was successfully completed and transformed into a PANI. The peak at 1114 represents the “electron like absorption”. All the discussion is for pure PANI. The peaks take place almost at the same position in case of Bi_2Te_3-PANI.

Electrometry involved taking measurements of the electrical conductivity related to nanocomposites using temperature. PANI is known to be an insulator as emaraldine base (EB) base. Doping turns PANI from insulator to conductor. PANI conductivity is largely reliant on the doping character. In this paper we doped PANI with Bi_2Te_3 and polymer to increase electrical conductivity. Although the carrier density increases with increasing temperature, the conductivity of PANI-nanocomposites remains low because of the low conductivity of Polyaniline.

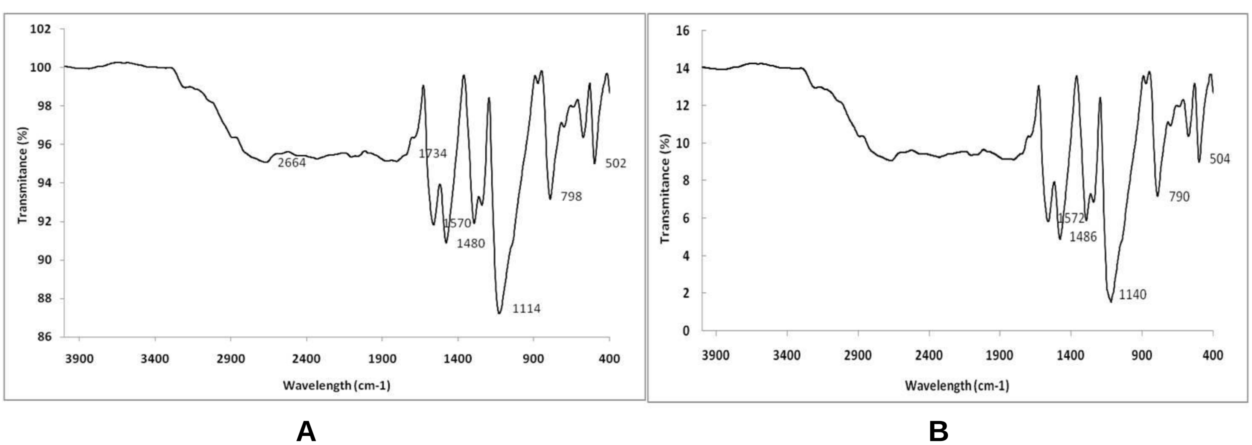

Fig. no. 2 - (A) FTIR spectra of pure PANI nanocomposite (B) FTIR spectra of Bi_2Te_3-PANI nanocomposite.

This is because nanofilter energy barrier was created in the polymer chain, now the charge carriers should overcome this barrier. But a slight initial increase in the PANI's conductivity can be observed as it contains Bi_2Te_3 inorganic compound.

Both the PANI and the Bi_2Te_3 increases the conductivity of electricity with temperature. The electrical conductivity of Bi_2Te_3-PANI over the entire temperature range is more in comparison to pure cells. This might be because of the generation of the molecular organization of Bi_2Te_3 in Bi_2Te_3-PANI complex formulation due to the large surface area that PANI provides to Bi_2Te_3.

WITH Se DOPING-

The connection between conduction rate and temperature can be observed from the figure. Bi_2Te_3-PANI sample's conductivity decreases with increasing temperature. As we increase the doping of Se, the conductivity fluctuates dramatically. The thermal conductivity gradually decreases to the lowest value. Significant loss in thermal conductivity is due to doping. The alloy scattering not only reduces the carrier mobility related to the sample, but also it increases the scattering effect on phonons, which considerably reduces the lattice thermal conductivity related to the product. In this stage, the thermal conductivity of the material comes largely from the input of the low-frequency photons. The grain boundary effect dispersion increased relatively and became large, which reduced the mean free path and thermal conductivity related to the photons.

Table no. 1- Comparison of the Room Temperature Transport Parameters (σ, S, κ, P, and ZT) between Bi_2T_3-PANI with and without Se doping under room temperature.

Composite	σ (S cm−1)	S(µ Vm−1 K−1)	P(µ Wm−1 K−2)	κ (Wm−1 K−1)	ZT
Bi_2Te_3-PANI without Se doping.	11.62	36.8	1.57	0.11	0.0042
Bi_2Te_3-PANI with Se doping	25.01	102.22	26.13	0.19	0.041

The thermal conductivity related to the substrate decreases with the enlargement of the amount of Se doping. On the one hand, the doping of Se would increase the carrier concentration, which leads to an increase in electrical conductivity. The above mentioned discussion helps in finding an inference that Se doping has a significant influence on Bi_2Te_3-PANI's morphology in a positive direction regarding the electrical conductivity.

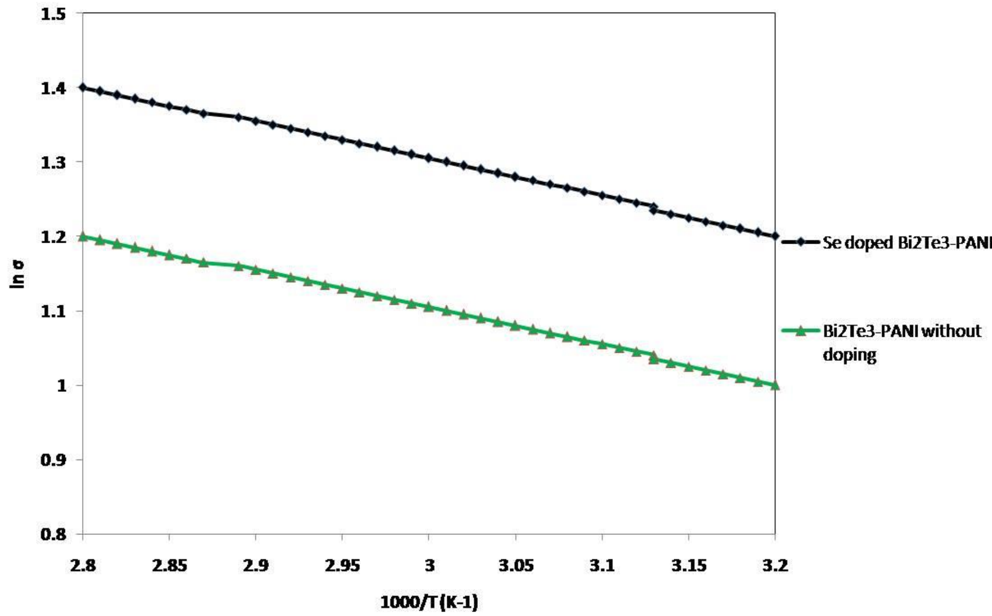

Fig. no. 3- The electrical measurements of Bi_2Te_3-PANI composite with and without Se doping.

From the Table no. 1 we can see the value of ZT increases with Se doping and we already have mentioned the efficiency of TE materials depends on ZT, we can clearly say that, Se doping has made Bi_2Te_3-PANI a better thermoelectric material.

Fig3 shows how the conductivity changes with respect to temperature. The conductivity increases with the doping.

CONCLUSIONS

The Bi_2Te_3-PANI was successfully synthesized and doped with Selenium. Increase in the three order magnitudes of conductivity have been observed after doping. Conduction mechanism has been explained. From thermal studies it is obvious that the conductivity increases with the rise in Se concentration followed by decline. The released enthalpy has been found to be associated with polymer metastability. It is quite clear from the studies that there are some structural changes of PANI after doping of Se due to which it becomes a better TE material, though it is quite clear from literature survey that conductivity depends on the amount of doping. Extreme amount of doping could lead to lesser conductivity.

As the amount of Se doping increases, the conductivity of the product after a certain large amount, decreases; the doping of Se element effectively decreases the thermal conductivity of the Bi_2Te_3-PANI than that of the undoped sample. Clearly a perfect amount of doping could lead to good quality of thermoelectric material.

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DO CONSUMERS BUY CONSCIOUSLY? INVOLVEMENT AND LEVEL OF IMPULSIVITY IN THEIR BUYING BEHAVIOUR

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Abstract: *The research investigates the concept of impulsivity widely spread in consumer psychology and covered dimensions from two disciplines i.e. economics and marketing. Moreover, it analyzed the consumers' propensity to make impulsive purchase decisions and the intensity of impulsivity across different product categories. It also focused on the relevance of disparities in consumer demographics (i.e., their gender, age, and income level) on their impulsive shopping behaviour in malls. The statistical procedures used in this research included "Frequency distribution, Percentage analysis, Mean score analysis, Cross-tabulation analysis, Independent Sample T-test, and One-way analysis of variance." Seventy-four percent of shoppers admit that they frequently buy things on the spot. Apparel, followed by consumables, footwear, grocery products, and personal care items, was found to be the most commonly purchased spontaneously and without prior planning in this survey. The study also observed that the differences in age and gender among consumers have a substantial impact on their propensity to make impulsive purchases, and buyers in the 18–25 years age range and female shoppers are most likely to encounter impulse purchases. Mall managers can leverage this finding to frame marketing strategies and economists may develop a new school of thought other than the "ideal consumer approach."*

Keywords: *Consumer Behaviour; Impulse Buying; Interdisciplinary; Rationality; Retail Store.*

1. INTRODUCTION

Marketers are indeed interested in understanding the buying tendency of consumers, which encompasses questions, i.e., what motivates them to make purchases in stores and what they think before, amid, and even after making the purchase. An ideal buying process comprises five phases: Recognition of a problem or need, the search for information, the evaluation of different options, the decision to purchase, and the behaviour after the purchase (as mentioned in fig. no. 1).

Fig. 1: Classical consumer buying behavior model

- Problem identification: Recognizing a need/desire driven by extrinsic and intrinsic stimuli.
- Browsing of Information: Searching for information to explore product-related alternatives. Salespeople, advertisements, Promotion, Customer memory etc. may offer such insights.
- Alternative evaluation: Alternatives are reviewed and analysed using predetermined criteria.
- Purchase decision: Product selection, followed by the act of making a purchase. Both can be differentiated from one another since they take place at different times and in other places.
- Post-purchase: Behavior of consumers after the completion of a purchase encompassing product consumption, post-purchase support, quality of service, gratification, and repeat purchases [17].

Properties of four distinct forms of consumer purchasing behaviour [14]:

- Ideal purchase behaviour: This buying behaviour relies on the theory of decision-making and rationality derived from the field of microeconomics. It outlines the presumptions of the rational buyer, who will be fully well aware and will hunt for the maximum benefits with a minor expenditure. The ideal customers do not exist, but it represents one end of a continuum of consumer patterns.
- Normative purchase behaviour: This approach reflects reality and implies that customers would plan, rationalise, critically compare, and decide on the amount of risk that suits them.
- Impulsive consumer purchase behaviour: This is the dominant buying behaviour and can be considered prevalent among consumers. Consumers try to control their inherent impulsive tendencies because they characterise unplanned purchases as normatively inappropriate and do not want to be viewed as childish or struggling to achieve behavioural control [24]. The dilemma is not deviating from the "typical" pattern of purchasing behaviour; instead, the problem is how most buyers act while making purchase decisions.
- Impulse purchase behaviour: This behaviour is the fourth category of customer purchasing patterns. The modern practice of impulsive shopping exhibits a trait of extreme behaviour, which, from a medical standpoint, is getting very close to meeting the criteria for a diagnosis of impulsive regulation disorder.

The negative connotation of impulse purchase tendency from the 60s has been updated to the extremes of consumer behaviour in today's market. Making impulsive purchases is not inevitable, but it is observed frequently in retail stores, malls, and health clinics.

The traditional premise of the purchasing/ decision-making process was grounded on the microeconomic approach of a fully conscious consumer making the best possible choices [9]. Microeconomic theory and decision science could only support the normative approach to buyer behaviour.

The shopper is often viewed as a "thinking machine" in decision science and microeconomics studies [16]. However, studies and reports from the marketing industry have shown that consumers often don't behave under the conventional framework. Perfect competition, in which all market participants act rationally, is a common pattern in theoretical economics. However, in practice, customers rarely have all they need (such as complete and accurate information, sufficient involvement, or sufficient drive) to make the "ideal" decision. Consumers can less objectively weigh the consequences of their choices when they are very engaged with a product which ultimately increases the likelihood of making an impulsive purchase. In reality, everyone makes impulsive purchases occasionally, while some do it more consistently than others. A concise antecedents model cannot explain this kind of consumer behaviour.

An impulse purchase is a psychological component of the complicated and dynamic functioning of the human mind. The term "self-regulation" describes the capacity to control one's emotional state and behaviour and to ensure that one conforms to the desired norm [28]. Consumers make irrational decisions because they lack the ability to self-regulate. When consumers consider purchasing on the spur of the moment, their impulsive urge functions as an influencer, while normative sanctions act as a restrainer. Consumers use several methods to justify their actions and to relieve any associated guilt or uncertainty [20].

We were propelled to investigate retail markets and explain customer behaviour by reporting these inconsistencies. A marketer must divide that base into narrower, more specific groups and provide each group with its own set of products and services to serve the needs of a broad customer base. So, the study proposed to investigate how customers' demographic disparities affect their propensity to make impulsive purchases and their shopping preferences towards various products.

Following a review of the relevant literature, the key variables were identified along with their operational definitions to build hypothetical applicability as follows:

Variable	Operational definition
Consumer purchase behaviour	Customer shopping patterns in malls and big retail stores [27].
Impulse Buying	A purchase made on the spur of the moment with no prior plan to purchase a specific product item or to complete a predefined purchasing task [2].
Demographic variables	The demographic variables examined for this research were age, gender, and income.

Table no. 1: Operational definitions of main variables

Consumers' propensity to make impulsive purchase decisions

There has never been any consistency between what they aim to buy and what they end up buying [11]. According to retailers, a certain amount of sales is attributed to "impulse buying," which is now seen as a subsection of typical buying patterns [5, 13]. Shoppers classify 30- 50% of their purchases as being made on the spur of the moment [3]. Impulsive shopping is a natural part of life and all shoppers have made nearly one impulsive purchase in their lifetime, and nine out of ten customers often make purchases based on impulse [29]. China, India, and Indonesia were the top three on the list of countries with the highest rates of impulsive purchasing and its early adoption. It is a component of lifestyles in western countries that cannot be avoided [25], expanding to the east without any particular border being defined [14].

Degree/Intensity of impulsivity across different product categories

The degree to which one is driven to purchase on the spur of the moment varies from product to product. Customers are more likely to make a quick purchase when the product includes certain features [13]. Earlier research reported that 27-62 % of consumers' chain store buying fell into the impulse buying category, yet some product lines remain unaffected by this behavior [3]. In spite of the fact that some things are more likely to be purchased on the spur than others, a specific collection of products is not a practical criterion for measuring impulsive consumer behaviour because of individual and cultural differences[28].

Relevance of Consumer demographic disparities (i.e., their gender, age, and level of income) on their propensity towards impulse purchase

Consumers' actions vary depending on their demographic profile, i.e., age, gender, income, level of education, etc. [7]. Over the past couple of decades, marketers have noticed that the connection of impulsive buying with various demographic features has gained significant relevance. It is believed to be a major factor in customers' final purchase decisions.

"Gender is a demographic factor which has been widely employed in these research areas [4]". Females, according to a research [6], are more prone to making spontaneous purchases than males. As per a survey [12], customers' impulsive purchases of apparel are affected by gender disparity. Men are less likely to engage in impulse purchases than women [1,15].

Consumers' purchasing habits change as per their age [26]. Also, shoppers' impulsiveness varies significantly with age [8]. In addition, as consumers' age increases, their propensity to engage in impulsive purchases declines [3]. Young people are more likely to make impulsive purchases, and their level of impulsiveness during shopping is higher than that of people of older age [1]. Generation Y shoppers are more likely to make immediate purchases [15].

The purchasing power of customers is proportional to their income, and those with more disposable income tend to make more impulsive purchases. Income also has a favourable effect on customers' Impulse buying behaviour. Individuals with higher incomes tend to make unplanned purchases more frequently than those with lower incomes [26, 30].

2. HYPOTHESES

In this paper, research is based on six initial hypotheses used to be validated or not during a specialised analysis:

H_1: The buying behaviour of customers is characterised by impulsivity.

H_2: The degree of impulsivity varies according to the product category.

H_3: Consumers' demographics (i.e., their gender, age, and level of income) significantly influence their propensity towards impulse purchase decisions.

H_4: Female shoppers are more inclined to impulse purchases than their male counterparts.

H_5: Young shoppers are more inclined to impulse purchases than older shoppers.

H_6: Shoppers with higher incomes are more inclined to impulse purchases than shoppers with lower incomes.

3. METHODOLOGY

The study's research methodology was classified into research design, sample design, questionnaire design, and data design.

3.1 Research design

The research design for this study was cross-sectional descriptive. The data collection was carried out in the field via a structured questionnaire. The months of May, April, and June 2022 make up the time frame for the study.

3.2 Sample design

According to the past researches, purposive sampling is frequently used in shopping malls for studying buying behaviour of consumers. So, the sample was selected using "Purposive Sampling technique". The research was based on the one-stage mall intercept survey method, which has been employed extensively in previous research to obtain data [2, 21]. The mall merchants in Rohtak city permitted us to conduct our survey inside their stores. Out of the total of 140 responses, only 100 were used for the study, primarily due to removing those who could not fill up all their information in the survey.

3.3 Questionnaire design

The scale used to evaluate the impulsive behaviour of buyers was adopted from previously conducted research [19, 31]. The responses were analysed using a Likert scale of five points, where 1 indicated that the respondent strongly disagreed and 5 suggested that the respondent strongly agreed. We asked shoppers (who had just finished checking out) to respond to a standardised questionnaire. The first question we asked them was whether they shop without making plans beforehand. In one section of the questionnaire, questions regarding the respondent's demographic profile, and in another, questions designed to assess the respondent's level of impulsive shopping behaviour were included.

3.4 Data design

The statistical techniques applied for this study were the "frequency" count, Percentage Analysis, Cross Tabulation Analysis, Independent Sample T-test, and ANOVA Technique.

4. DATA ANALYSIS AND DISCUSSION

Demographic Catagories	**Unplanned/ Impulsive buying**		
	YES	**NO**	**Total**
Below two lakhs	26	13	39
Two lakhs-Five lakhs	32	4	36
Above five lakhs	16	9	25
18-25	30	6	36
25-35	24	10	34
35-45	20	10	30
Male	34	16	50
Female	40	10	50
Total	74	26	100

Table 2.1: Demographic profile and categorization of buyers based on impulsiveness

Impulse buying decision Indicators	Mean Score
When I go shopping, I purchase things I hadn't intended to buy.	3.28
I am one of those who make unplanned purchases.	3.01
When I find something interesting, I don't consider the consequences and buy it.	3.07
It is fun to buy spontaneously.	3.24
I avoid buying anything that is not in my shopping list.	2.76
I regret after buying that I didn't intend to purchase.	2.79

Table 2.2: Mean Scores of Customers' Perception of Impulse Buying

Fifty men and fifty women constituted the total sample of the survey. There are 36 people between the age of 18 and 25, 32 between the age of 25 and 35, and 32 between the age of 35 and 45. Among those who participated, 39 had annual earnings of less than two lakhs, 36 had annual income between 2 lakhs and five lakhs, and 25 had annual income of more than five lakhs. Most respondents (74%) act impulsively when it comes time to make their purchasing decision, whereas just 26% of shoppers make their decisions after giving it some thought. The findings reveal that most customers engage in impulse buying, which suggests they do not follow the rational or traditional buying process (Table 2.1).

Women (N=40) are more prone to exhibit impulsive behaviour than men (N=34). It was found that shoppers between the ages of 18 and 25 (N=30) are more likely

to make impulsive purchases than those of older generations (N=24 & 20, respectively). The data also reported that the consumers with the highest income bracket are the least likely to indulge in impulse shopping (Table 2.1). The shoppers admit that they frequently walk out of stores with items they did not intend to buy. As their responses to the first four questions are significantly higher than the average value (i.e., 2.5). Whenever buyers like something, they decide to purchase it and they are reckless about the repercussions of their actions. However, they disagreed with the assertions that "they avoid things that are not on their shopping list and that they regret after shopping without a plan" because these two statements are somewhat close to the average value (Table 2.2). The mean indicated that consumers act impulsively while making purchase decisions and do not even consider the outcomes of their purchases after making them. They take pleasure in acting on impulses whenever they shop without a list, and they are not likely to have feelings of regret due to such purchase decisions.

Therefore, the hypothesis (H_1) "The buying behaviour of customers is characterised by impulsivity" was accepted. However, a subset of customers who shop thoughtfully can keep their minds away from wandering and resist being tempted to things that were not on their shopping list. It is widely acknowledged that impulsive purchasing is inherent in consumer lifestyles and is responsible for selling a significant percentage of goods in a diverse array of product categories [22].

Products	Ranks
Apparels	1
Footwear	2
Eatables	3
Cosmetics	4
Grocery	5

Table 3: Preferences for products purchased impulsively by buyers

There is no uniform level of impulsivity in customers' purchase behaviour among the different types of products. Out of the above products, apparel is the one that is purchased the most, followed by footwear, edibles, cosmetics, and groceries (Table 3). Therefore, the hypothesis (H_2) "The degree of impulsivity varies according to the product category" was accepted.

Customers explore apparel outlets to obtain some idea of the most recent trends in the industry rather than depending on their shopping list [19]. In that case, there is a high chance that they may make impulsive purchases. In addition, the desires of individual customers are likely to fluctuate more extensively than their essential requirements. This implies that clothing as a product class may serve as a stimulus that influences impulsive buying.

Gender	Male	Female
N	50	50
Mean	2.7700	3.2633
Std. Deviation	.74069	.73316
t-value	3.347	
Significance value	.001	

Table 4: Summary of Independent Sample T-Test

The respondents were given a scale ranging from 1 to 5 for evaluating their perception regarding impulse purchase behaviour; the possibly average attribute value is 2.5. Based on the findings, the values of all measures of impulsive buying for both male and female customers are much higher than the mean. The significance of gender differences in impulsive purchases is further supported by the standard deviation, which describes the dispersion of the random value around the mean. When the coefficient is close to 1, the effect is strong. The standard deviation values are all relatively close to 1. There is a difference between the means of male consumers and female consumers (Table 4). Female consumers (mean 3.26, Std. deviation .73) exhibited higher impulsivity than male consumers (mean 2.77, Std. deviation .74).

Hence, the hypothesis (H_4) “Female shoppers are more inclined to impulse purchases than their male counterparts” was accepted. This finding is consistent with some studies [8, 23] that women tend to make more impulsive purchases than men because of their extreme sensitivity to advertising and natural inclination towards visually appealing products.

The t-test result reported that the impulsive purchases made by male customers differ considerably from those made by female customers (p-value < 0.05). The hypothesis "Consumers' gender influences their propensity towards impulse purchase decisions significantly." was supported. These results show consistency with a research [13] that consumers' propensity to make spontaneous purchases differs by gender. On the other hand, one research reported arguments of non-significant variations between men and women when it came to making impulsive purchases [4].

Age	18-25	25-35	35-45
N	36	32	32
Mean	3.2031	3.1615	2.7222
Std. Deviation	.73247	.80931	.70296
f-value		4.373	
Significance value		.015	

Table 5: Summary of One-way ANOVA

There is a difference in the average value of consumers between those aged 18 to 25 and those aged 25 and above (Table 5). According to the findings, younger shoppers were found to exhibit higher levels of impulsivity (mean 3.20, standard deviation .73) than older shoppers (mean 3.16, standard deviation .80 and mean 2.72, standard deviation .70 correspondingly). Therefore, the hypothesis (H_5) “Young shoppers are more inclined to impulse purchases than older shoppers significantly” was supported. This finding is similar to a couple of studies [1,8] that there was a negative correlation between customers' age and their propensity for making impulsive purchases. Hence, young buyers are more susceptible to impulsive purchases than older buyers.

Impulsive decisions made by younger customers are significantly different from those made by consumers of older ages (p-value less than 0.05). Therefore, the hypothesis that "Consumers' age significantly influences their propensity towards impulse purchase decisions" was accepted. This study is in line with other surveys [4, 8, 13], implying that the Impulse purchase behaviour of customers differs significantly among age groups.

Income	**Below two lakhs**	**Two lakhs-Five lakhs**	**Above five lakhs**
N	39	36	25
Mean	2.9658	3.1204	2.9467
Std. Deviation	.87374	.66739	.76636
f-value		.504	
Significance value		.605	

Table 6: Summary of One-way ANOVA

Shoppers with incomes in the range of 2 lakh-5 lakh had a higher propensity (Table 6) to make impulsive purchases (mean 3.12, standard deviation .66) as compared to other groups of shoppers (mean 2.96, standard deviation .87 and mean 2.94, standard deviation .76 respectively). So, the hypothesis (H_6) “Shoppers with higher incomes are more inclined to impulse purchases than shoppers with lower incomes” was not accepted. In contrast, some argumentative studies [4, 26, 30] stated that those with higher incomes tend to be more prone to making impulsive purchases.

Since the p-value is higher than 0.05, the findings of the ANOVA do not indicate any significant relationship between one's income and their tendency to make impulsive purchases (Table 6). So, we can conclude that the hypothesis “Consumers' income influences their propensity towards impulse purchase decisions significantly” was not supported. This finding is consistent with a former study [15] that no relationship exists between a consumer's income and their propensity to make impulsive purchases. However, the study's results contradict other research findings [4] that household income has a considerable influence on people's tendency to make impulsive purchases. As shoppers’ age and gender affect significantly impulse buying but income level doesn’t have any significant effect on their impulsive decision, Hypothesis (H_3) was partially accepted in the study.

5. CONCLUSION

The normative approach of microeconomics is not entirely applicable to the market. The heartbeat of impulse purchases is alive and thriving. The research proposed a novel viewpoint, a reigniting typology of consumer purchasing patterns, and a shift in the socially accepted standards of being a shopper in the present day. Attracting shoppers and sustaining loyalty in today's increasingly competitive retail market is a tough challenge. This study confirmed the significance of the impulsive decision in making a purchase. It revealed various demographic traits of people who tend to make purchases on the spur of the moment. This finding has provided a light for businesses focusing on specific age groups or gender categories while chasing customers. Promoting the "less-impulsive" classes is essential to strike a better balance. Companies must be aware of the behavioral differences among various demographic groups. Retailers must adopt a specific strategy for each category group to maximise productivity and sales in their stores. We feel that the summarised findings offer substantial implications that could promote the scientific understanding and retail applicability of this critical topic in consumer research. The study's implications could be helpful for practitioners, economists and academics in specifying criteria and designing a strategic framework to identify and categorize consumers based on their impulsivity trait.

6. RESEARCH IMPLICATIONS

This study provides marketers and mall owners with some significant insights. Many management initiatives might be taken to enhance the consumers' purchasing activity in retail stores. The relevant managerial implications emerged from the study findings are mentioned below:

(1) Sales managers need to be more engaged in offering promotional activities to shoppers, as most buyers would be motivated to purchase impulsively by these programs.

(2) Focusing upon the fact that gender strongly influences impulsive purchase behavior, retailers might build customized programs for distinct gender groups of shoppers.

(3) Concentrating on the observation that age greatly affects impulsive purchasing patterns, merchants must develop customized programs targeted to the diverse age group of buyers.

(4) Depending on the research outcomes that most of the shoppers who referred to the impulsive segment were females, merchants may build up-scale pictures of the store to persuade this group of shoppers.

(5) The survey's findings reveal that most impulse buyers are young people; stores could be decorated with trendy art and create special incentives to cater to this segment and encourage their sudden purchase decision.

(6) Since apparel was mostly purchased on impulse, stores need to develop their promotional strategies to induce apparel customers to buy on the spot.

7. LIMITATIONS AND FUTURE RESEARCH DIRECTIONS

The limitations of any study determine how far its results can be generalised. While this study has the potential to significantly contribute to our understanding of consumer behaviour and impulse purchases, it also has some significant drawbacks. Because of the small sample size, inductive generalisation is a problem for this study. The views of urban people may differ from those living in rural areas. Although the findings only include shoppers in malls, they can be extrapolated to consumers who shop at ordinary brick-and-mortar establishments. More research is required to determine if other factors affect Indian consumers' propensity to make unplanned purchases. Investigation is necessary to gain a deeper understanding of the factors that influence impulsive buying, including a diverse sample of respondents (from various geographic locations and cultural backgrounds).

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RELATIONSHIP BETWEEN ANXIETY, INTOLERANCE OF UNCERTAINTY & SELF-CONCEPT CLARITY: A STUDY ON INDIAN YOUTH

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Abstract: *Anxiety is a feeling of edginess and vexation, usually generalized and distributed as an exaggerated reaction to a situation that is subjectively seen as alarming. Intolerance of uncertainty is the tendency to react negatively to unknown situations and experiences on an emotional, cognitive, and behavioral level. The degree to which self-concept beliefs are explicitly and confidently established, internally consistent, and stable over time is referred to as self-concept clarity. The youth today faces many challenges in terms of career advancement, relationship coping struggles, competition, peer pressure, body image, and appearances, among other things. In addition to socio-environmental, personal, and psychological factors, the mental and physical health of the youth is also significantly impacted. The present study focused on the relationship between anxiety, intolerance of uncertainty and self-concept clarity among the emerging adults. It was found that anxiety was positively correlated with intolerance of uncertainty (r=.805, α= .01) and negatively correlated with self-concept clarity (r=-.814, α= .01); while intolerance of uncertainty was found to be negatively correlated to self-concept clarity (r=-.821, α= .01). The present study would thus help explore different facets about youth, they way they cope with anxiety, the mechanisms and reference frameworks they use to evaluate themselves and while doing all this; how comfortable they are with the ever-changing scenarios of life.*

Keywords: *Anxiety, Intolerance, Uncertainty, Self-concept, Youth*

1. INTRODUCTION

Often convoyed by nervous behavior, anxiety is an emotional state marked by an unpleasant state of inner upheaval. Anxiety is a feeling of edginess and vexation, usually generalized and distributed as an exaggerated reaction to a situation that is subjectively seen as alarming. It is oftentimes accompanied by muscular tension, uneasiness, weariness and difficulty in concentration. Anxiety is often closely associated with fear but the two are not synonymous. Fear is a response to a actual or sensed proximate danger; while anxiety involves the anticipation of a future threat. People experiencing anxiety may retreat from settings or conditions which have given rise to anxiety in the past. Anxiety can be experienced with prolonged and persistent symptoms that deteriorate the quality of life, also referred to as chronic (or generalized) anxiety. On the other hand, it can also be experienced in short surges with unpredictable, exhausting and draining panic attacks, called acute anxiety.

Symptoms of anxiety may vary in their nature and impact from one person to another. Anxiety can cause physiological, psychological, behavioral, emotional and cognitive effects on an individual which may include neurological, digestive and respiratory problems; the tendency to harm oneself; avoiding the situations that have proven to be unpleasant in the past; feelings of dread or apprehension and; thoughts about the suspended dangers in one’s environment.

Intolerance of uncertainty is the tendency to react negatively to unknown situations and experiences on an emotional, cognitive, and behavioural level. Intolerance of uncertainty is considered to be a dispositional trait that stems from a set of negative views about uncertainty and its repercussions, as well as a proclivity to react negatively to uncertain circumstances and occurrences on an emotional, cognitive, and physiological level [1] (Buhr & Dugas, 2009).

2. LITERATURE REVIEW

The research in the field of intolerance of uncertainty began a few years prior to the development of Dugas et al. (1998) model of GAD. [2] At that time, there was a large body of research on a concept called "Intolerance of Ambiguity" (IoA), which was coined fifty years ago by Frenkel-Brunswick (1948) [3]. This term was coined to describe someone's predisposition to see ambiguous situations as dangerous and to respond with discomfort and avoidance when confronted with new or difficult problems that cannot be solved [4, 5] (Budner, 1962; Grenier, Barrette, & Ladouceur, 2005). Intolerance of uncertainty is a complex mental process which is considered to be comprising of three key components:

- Positive beliefs regarding worrying: This kind of positive beliefs lead to the idea that worrying is beneficial in some way. As if worrying can help an individual to achieve certainty in unpredictable life situations.
- Negative problem orientation: Negative problem orientation occurs when the individual feels unable to solve problems, perceives problems as daunting or as hurdles or limitations, and has doubts about his potential to resolve complications
- Cognitive avoidance: Cognitive avoidance refers to the practice of only coping with issues when they are truly unavoidable.

Individuals who suffer from anxiety, particularly those who worry excessively as adults, appear to be more intolerant of uncertainty and will always attempt to forecast and prepare for anything in order to avoid or eliminate it.

The degree to which self-concept beliefs are explicitly and confidently established, internally consistent, and stable over time is referred to as self-concept clarity. Self-concept clarity is one of a cohort of constructs that concentrate on the self-structural concept's aspects. Self-concept clarity is a reliable indicator of self-assurance. In addition, self-concept clarity is significantly connected to the execution of concrete identity decisions, while it is negatively associated with identity crises resulting from the reconsideration and abandonment of current commitments.

Some people have a strong sense of understanding regarding who they are and where they want to go in life. They understand their own strengths and limitations, as well as the essence of their personalities and their positions on critical attitudes and values. On the other hand, some people may have less defined self-concepts. These people may not be secure of who they are, may not know where they stand on important topics, and may be unsure of their abilities. Self-concept clarity refers to the degree to which people with a strong self-concept are aware of who they are, do not hold contradictory views, and maintain consistent perspectives over time. As opposed to self-esteem, which is seen as an overall assessment of the self as good or poor, self-concept clarity is the way people's information about themselves is cognitively structured.

2.1 Anxiety and intolerance of uncertainty

Intolerance of uncertainty has been identified as a critical process variable in the treatment of generalized anxiety disorder. A number of professional practitioners have previously employed intolerance of uncertainty as a point of intervention for a wide range of anxiety disorders, including generalized anxiety disorder and obsessive compulsive disorder. There are three plausible explanations for the relationship between anxiety and intolerance of uncertainty. The first issue that must be addressed to understand this relationship is the lack of agency (i.e., a sense of uncontrollability). The second element to consider is the inevitability of negative effects in the future. The third aspect to consider is the occurrence of events that are undeniably certain but have not yet been recognized as potentially dangerous (Carleton et al, 2014) [6]. Although both common and particularized susceptibilities can contribute to the emergence and retention of anxiety disorders, a large number of psychopathology models suggest that intolerance of uncertainty (IU) is a central feature of anxiety-related encounter (Shihata et al, 2016) [7]. It is also considered that a high IU level causes in an increased identification of prospective difficulties and a negative problem orientation (Dugas et al., 1997) [8], both of which have been associated to generalised anxiety disorder. Numerous studies have found a significant connection between IU and symptoms of anxiety disorders, mood disorders, and OCD [9-13] (Freeston et al., 1994; Steketee et al., 1998; Clark, 2002; Tolin et al., 2003; Dugas et al., 2007). Some studies, on the other hand, have found a more specific link between IU and GAD, which is consistent with the idea that IU contributes to the disorder's unique origin and clinical presentation.

2.2 Anxiety and Self Concept Clarity

Self-concept Clarity draws attention to an organism's coherence of identity, how certain one is in one's traits, and to what extent are these traits consistent and stable. Lack of self-concept clarity has been linked to poor psychological adjustment and functioning, which may result in a condition of continual uneasiness and doubt consequently paving the way for a variety of psychological anomalies, including depression, anxiety, and autism.

Higher degrees of self-concept clarity, on the other hand, have been connected to adaptive psychological adjustment and functioning [14] (Lodi-Smith, J., & DeMarree, K. G., 2017). In a study conducted on 235 participants aiming at examining the impact of self-concept clarity and intolerance of uncertainty on the symptoms of generalized anxiety disorder (GAD), it was found that elevated intolerance of uncertainty and lower self-concept clarity emerged as unique correlates of high GAD symptoms [15] (Kusec et al., 2016). Self-concept clarity has also been connected to other aspects of anxiety, such as social anxiety and adult separation anxiety disorder. According to some theories, there is a substantial probability that social anxiety disorder is produced by unfavourable mental conceptions of the self-paired with a fear of exposing these core beliefs of the self [16] (Rapee & Heimberg, 1997). People with low self-concept clarity are more likely to develop social anxiety disorder because external stimuli are more likely to alter their self-concepts. Self-concept clarity has been found to have a substantial negative relationship with social anxiety, implying that people who have low self-concept clarity have higher levels of social anxiety [17] (Krupa, 2018). With a lot of studies focusing on the fact that self-concept clarity influences the anxiety levels in individuals, some researches have also focused on the fact that existing anxiety may also affect one's self-concept clarity. In a laboratory experiment conducted by Orr & Moscovitch (2015) [18], it was found that manipulations aimed at reducing the participants' self-concept clarity were only successful with the participants who had already high levels of social anxiety.

2.3 Intolerance of uncertainty and self-concept clarity

Individuals with higher levels of self-concept clarity have a strong sense of understanding regarding who they are and where they want to go in life. They understand their own strengths and limitations, as well as the essence of their personalities and their positions on critical attitudes and values which makes them prepared to deal with the unknown situations life may put forth for them as well as to deal with them effectively thus indicating an antagonistic relationship between the two.

In a study conducted on 166 university students, significant negative correlation was found between self-concept clarity and intolerance of uncertainty thus strengthening the fact that being aware of one's self can make a person face unpredictable life situations with greater efficiency [19] (Butzer & Kuiper, 2006). A study conducted on 235 participants indicated that high levels of intolerance of uncertainty and low levels of anxiety were the key correlates of generalized anxiety disorder [15] (Kusec, Tallon & Koerner, 2016). Several studies have highlighted a low self-concept clarity and high intolerance of uncertainty as factors hindering personal growth while higher levels of self-concept clarity have been associated with positive mental states and life stances [20-22] (Leite & Kuiper, 2008; Cicero, 2017; Birrell et al, 2011).

3. METHOD AND MEASUREMENTS

The participants comprised of 300 college and university students including both males and females. A purposive sampling technique was used for the same.

3.1 Intolerance of Uncertainty Scale, Sh ort Form (IUS-12)

The IUS-12 was developed by Carleton and colleagues (2007) [23] as a 12-item short-form of the original 27-item Intolerance of Uncertainty Scale [9] (Freeston et al., 1994), which assesses responses to uncertainty, ambiguous situations, and the future.

3.2 The Self-Concept Clarity Scale (SCCS)

The self-concept clarity scale tends to assess the extent to which self-beliefs are stable, clearly stated and confidently defined. The self-concept clarity scale is a 12 item uni-dimensional self-report measure of the temporal stability, consistency and clarity of self-beliefs [24] (Campbell et al., 1996)

3.3. Beck Anxiety Inventory (BAI)

The Beck Anxiety Inventory, developed by Beck, Epstein, Brown, and Steer [25] (1988), is a 21-item multiple-choice self-report inventory used to assess the severity of anxiety in children and adults.

4. RESULTS AND DISSCUSION

The synthesis of research' s results is described in the tables 1 and 2, and discussion part of the paper's text clarifies some scientific validation/invalidation

Table 1: *Descriptive Statistics (N=300)*

	Mean	S.D.	Min	Max
SCC	36.48	7.07	21	58
IU	34.52	7.63	14	60
Anxiety	26.06	11.03	2	54

SCC- Self-concept clarity; IU- Intolerance of Uncertainty

Table 2: *Correlational Analysis*

	SCC	IU	Anxiety
SCC	1	-.821**	-.814**
IU		1	.805**
Anxiety			1

**Significant at .01 level

It was evident from the obtained data that the participants reported average levels of self concept clarity and intolerance of uncertainty while slightly below average level of anxiety was reported. Correlation analysis indicated that both anxiety and intolerance of uncertainty are negatively correlated to anxiety while a positive correlation was found between intolerance of uncertainty and anxiety.

The objective of the present study was to assess the relationship between anxiety, intolerance of uncertainty and self-concept clarity among university students. The results revealed that anxiety was positively correlated with intolerance of uncertainty while it was negatively associated with self-concept clarity. Intolerance of uncertainty was also found to be negatively associated with self-concept clarity while a positive association was found between anxiety and intolerance of uncertainty. The results are discussed in detail as follows:

4.1 Anxiety and Self-concept clarity

Results (table 2) showed that anxiety and self-concept clarity have a significant negative correlation (r=-814, p<.001). While self-concept clarity makes a person accept his shortcomings and have faith in his abilities, anxiety makes him doubtful about his efficiency to deal with upcoming life events. High levels of self-concept clarity can enhance adaptiveness and functional flexibility while lower levels of self-concept clarity may pave way for problems like anxiety and depression [14] (Lodi-Smith, J., & DeMarree, K. G., 2017). Emerging adults belong to an age group that is still exploring their identities, strengths and weaknesses; those who can clearly put a finger on who they really are or want to be are more open to failure, constructive criticism and a meaningful feedback while the ones who cannot separate their real selves from their ideal selves or deny their potentials and shortcomings can never be prepared enough for the unforeseen in life and will always be nervous and on edge when it comes to handling situations.

4.2 Anxiety and Intolerance of Uncertainty

Results (table 2) showed that anxiety and intolerance of uncertainty have a significant positive correlation (r=805, p<.001). Inability to perform with optimum potential and efficiency under sudden or novel situations is one of the consequences of anxiety. Those who cannot bear uncertainty and sudden changes in their daily lives and routines are also found to be uneasy, unable to function and display changes in their emotional and mood states while dealing with something new. A high IU level causes in an increased focus on possible difficulties and a negative problem orientation [9] (Freeston et al., 1994), both of which have been associated to generalised anxiety disorder. Numerous studies have found a significant connection between IU and symptoms of anxiety disorders, mood disorders, and OCD [9-13] (Freeston et al., 1994; Steketee et al., 1998; Clark, 2002; Tolin et al., 2003; Dugas et al., 2007).

4.3 Intolerance of Uncertainty and Self-concept clarity

Results (table 2) showed that intolerance of uncertainty and self-concept clarity have a significant negative correlation (r=-.821, p<.001). It has been known from numerous studies that intolerance of uncertainty may lead to excessive worry and maladjustment while self-concept clarity enables an individual to stay stable in face of any unforeseen situation as the person has an accurate understanding of his potentials and the resources at hand. It has also been found that individuals with higher levels of intolerance of uncertainty often indulge in social comparisons and are not content with the things at hand thus further promoting a lack of mastery on one's life while higher levels of self-concept clarity would do the opposite [19] (Butzer & Kuiper, 2006).

5. CONCLUSION

The findings of the present study indicate that higher levels of self-concept clarity can help reducing the impact of anxiety and intolerance of uncertainty on emerging adults. Programs focusing on inculcating mind-sets and behaviours that promote self-acceptance can be useful in alleviating or minimizing the impact of anxiety as well as will promote tolerance among the youth.

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